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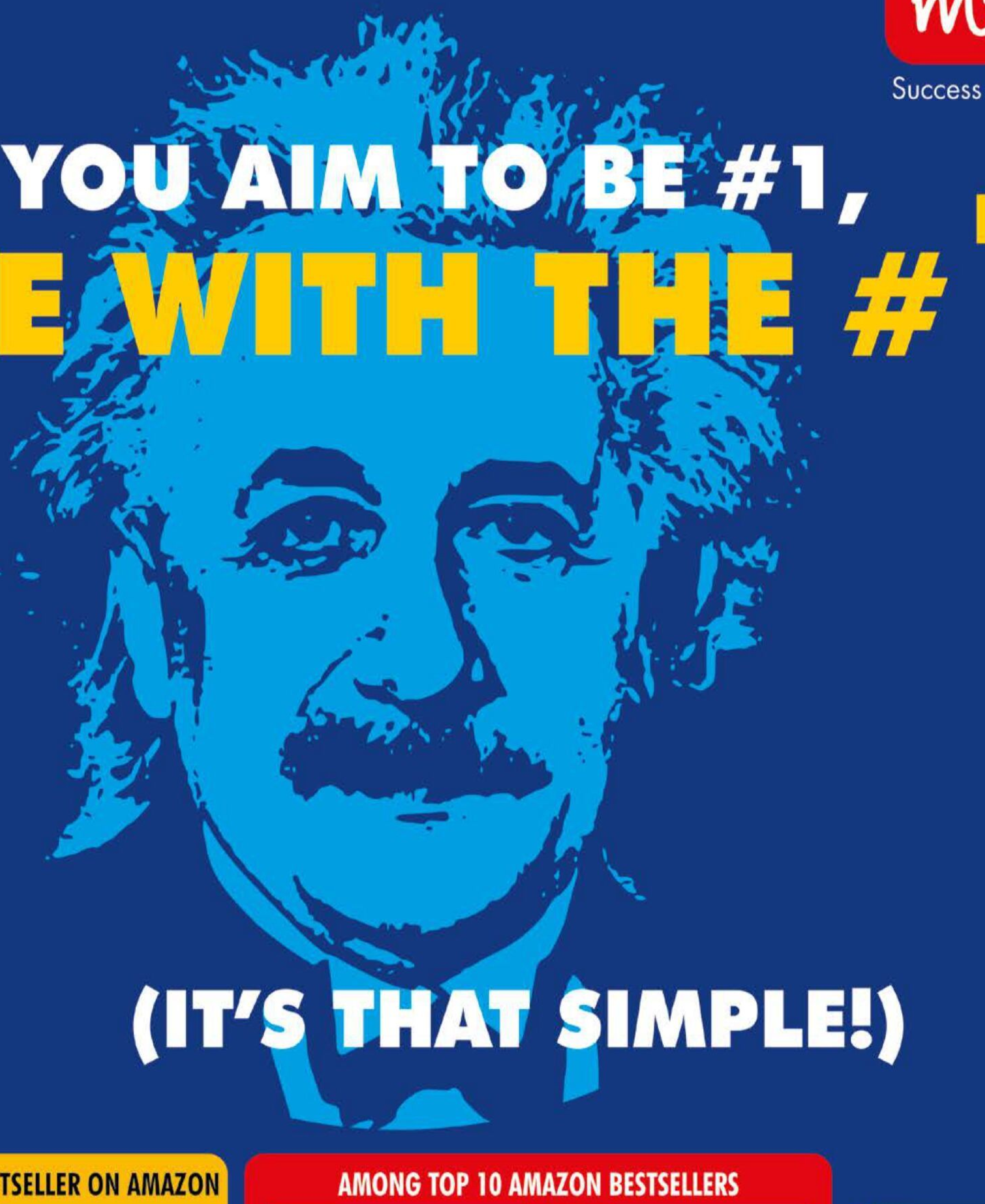

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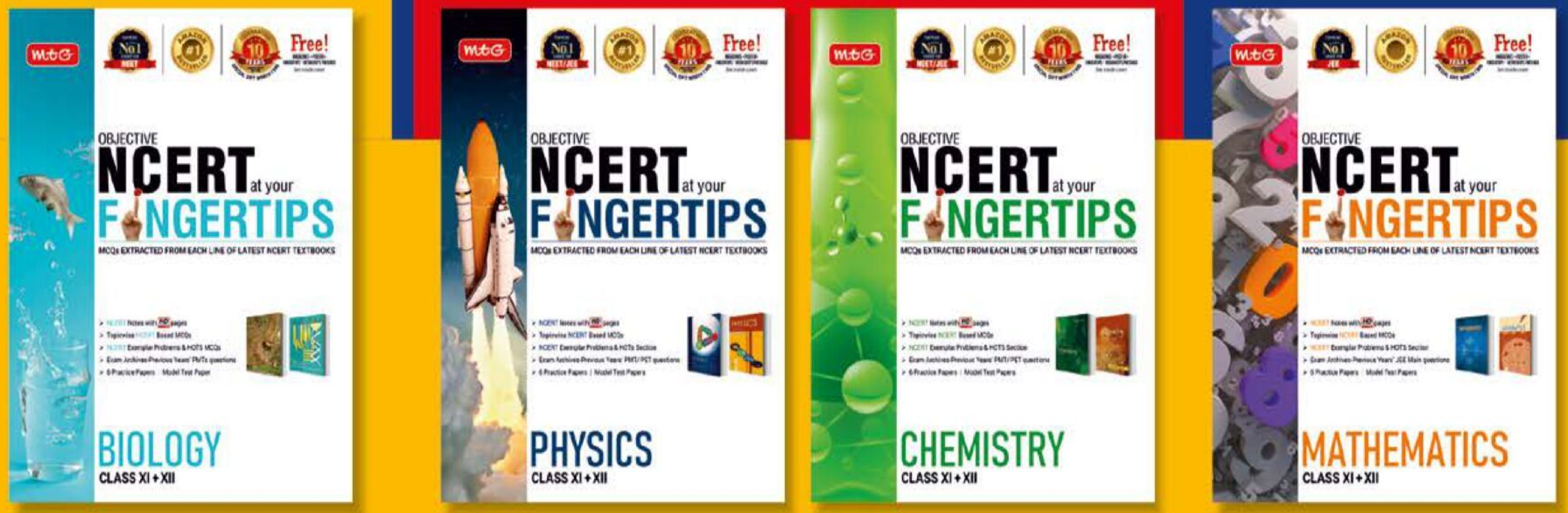

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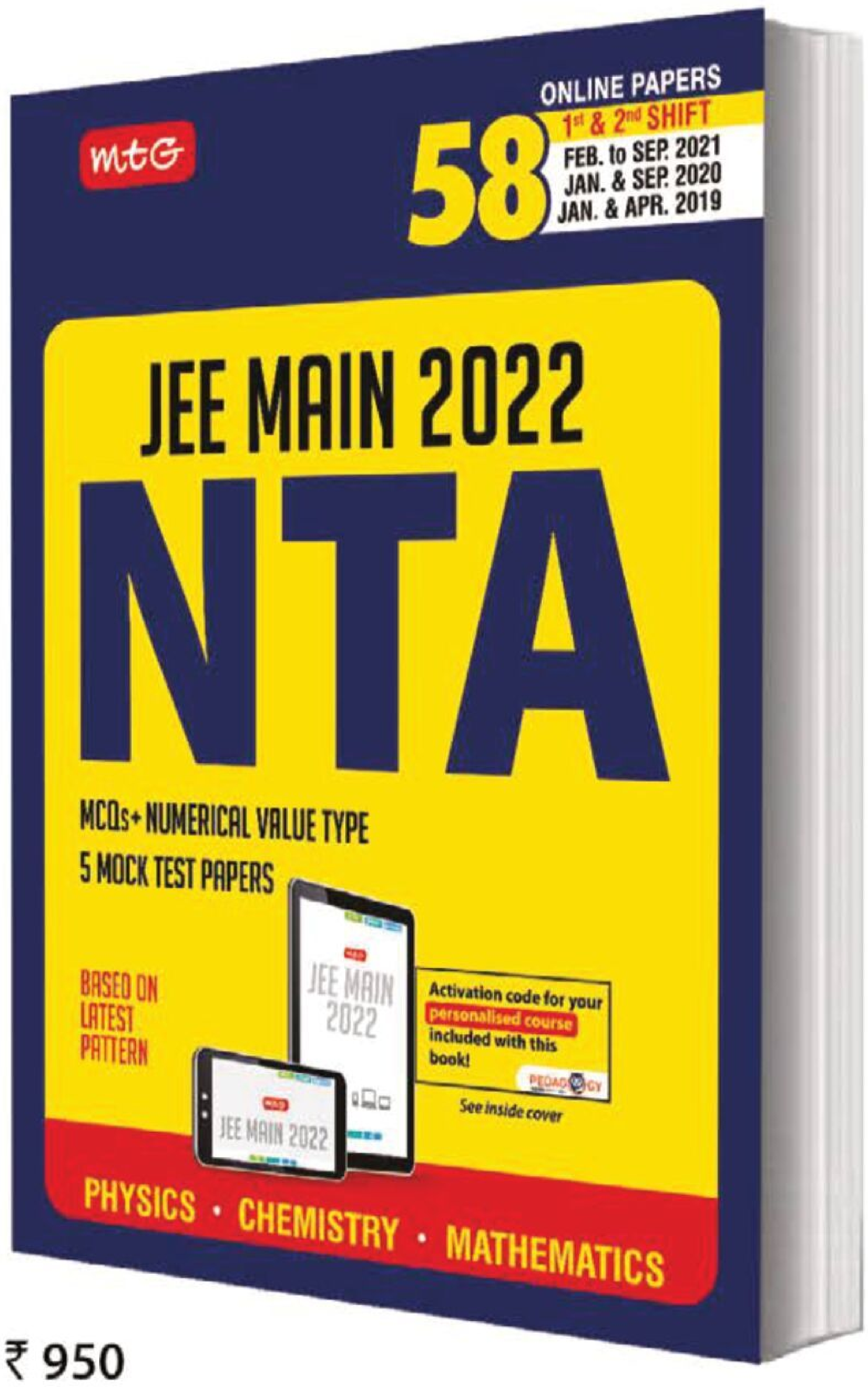
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An organic way to learn Inorganic Chemistry

Keep these tricks up your sleeve to master Inorganic Chemistry, one of the most scoring subjects in JEE and NEET exams.

Believe it

Inorganic Chemistry is the secret weapon to score more in JEE and NEET exams. The general practice among many students is to pay less attention to Inorganic Chemistry as it requires a lot of "mugging up". Agreed that its preparation does require dedicated time and efforts, but the result will be well worth the efforts.

Also know this

The knowledge from this subject often overlaps or is used in other chapters including several reactions in Mole Concept, Chemical and Ionic Equilibrium, Electrochemistry, Oxidation and Reduction and complete Organic Chemistry. Hence, Inorganic Chemistry can be an aid that defines your performance in Chemistry as a whole.

While there are several aspects of the subject that you have to memorise, you also need to have some level of understanding. You need to know the trends across the periodic table which encompasses similarities between several properties of elements and various reactions. In fact, some topics like Electrochemistry, Chemical Bonding, Solutions and Colligative properties are based entirely on the trends and properties originating in Inorganic Chemistry.

Bond Formation, Bond Order, Overlapping and Hybridisation discussed in Chemical Bonding also play a vital role in almost every organic reaction. The concept of Vacant Orbitals and Lone Pairs is helpful in studying different intermediates of Organic Chemistry. Halogenation is a major reaction of Organic Chemistry which is solely dependent on reactivity of halogens related with p-block.

Let's look at a few tips that might guide you to ace both JEE and NEET exams.

Be well-versed with NCERT

The primary way to prepare for Inorganic Chemistry for JEE and NEET exams is to learn each and everything from NCERT. You might be uninspired to study certain topics, but you mustn't give in. Follow a few simple steps to make future revisions of these topics easier.

- Highlight or underline the important sections, so you don't have to read every line during revision.
- Make your own notes. You can also opt to study from notes created by expert for this subject. This will ensure that you do not miss topics that might be significant from the exam point of view.
- An added benefit of NCERT is that explanation of almost every important topic is followed by examples. If you are unable to solve these, re-read and understand the concepts of the problem once more. Additionally, NCERT chapter-ends have exercises consisting of easy to advance level questions that are invaluable. Don't miss even a single problem from these exercises.

Use mnemonics or learn by writing

It is not uncommon for students to memorize Inorganic Chemistry through mnemonics — using songs, imagery or abbreviations to remember the subject efficiently. However, if mnemonics is not your cup of tea, you can go the old school way — take notes.

Don't start late

Some students take up Inorganic Chemistry at a much later date. Grasping a vast amount of information in a relatively short span of time is not advisable as the other dependent topics will end up being difficult to comprehend. Hence, make sure to start with this subject early on. Do not spend too much time on it, but just enough to build a strong foundation of all the topics.

Solve lots of questions

Practice, practice and practice those questions again. Our brain processes information better when it goes through the data repeatedly. When you attempt tricky questions, you take different approaches to get to the right answer.

Use notes for revision

One cannot possibly go through the entire NCERT syllabus in the run-up to the exam. This is where notes, especially short notes, come in. You can quickly revise a significant part of the syllabus in a short amount of time without exertion. Hence, maintain a small notebook for Inorganic Chemistry, with your notes being as concise and organised as possible.

Although perceived as one of the difficult subjects, Inorganic Chemistry can in fact prove to be beneficial if studied with sincerity and determination. Put up the periodic table at your study desk. Read and repeat the names, atomic numbers, symbols and atomic weights regularly. In fact, the mere act of 'seeing' the periodic table can help you to retain the information, as repetition of visual or auditory memory is known to translate into long term memory.

Remember, nothing is impossible if you put your mind to it, and in this case memory too.

Disclaimer: The information provided in this article is of a general nature and should not be considered a substitute for professional advice.

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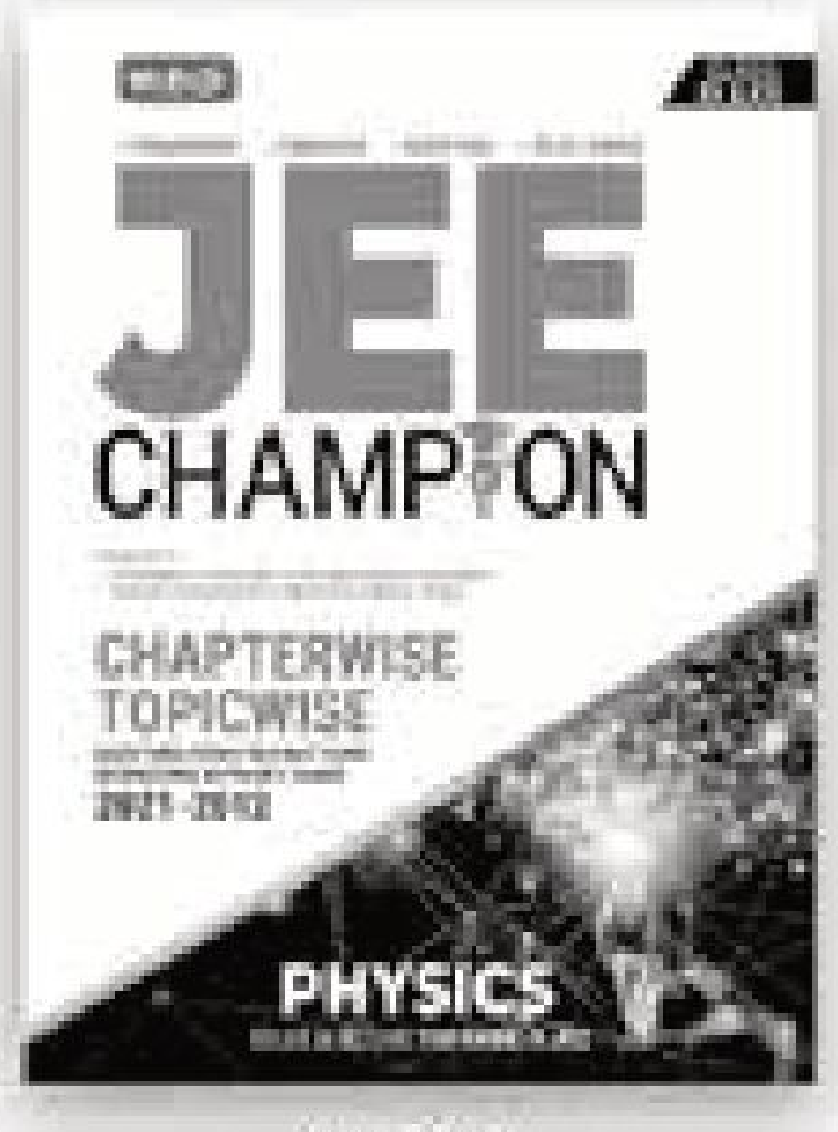
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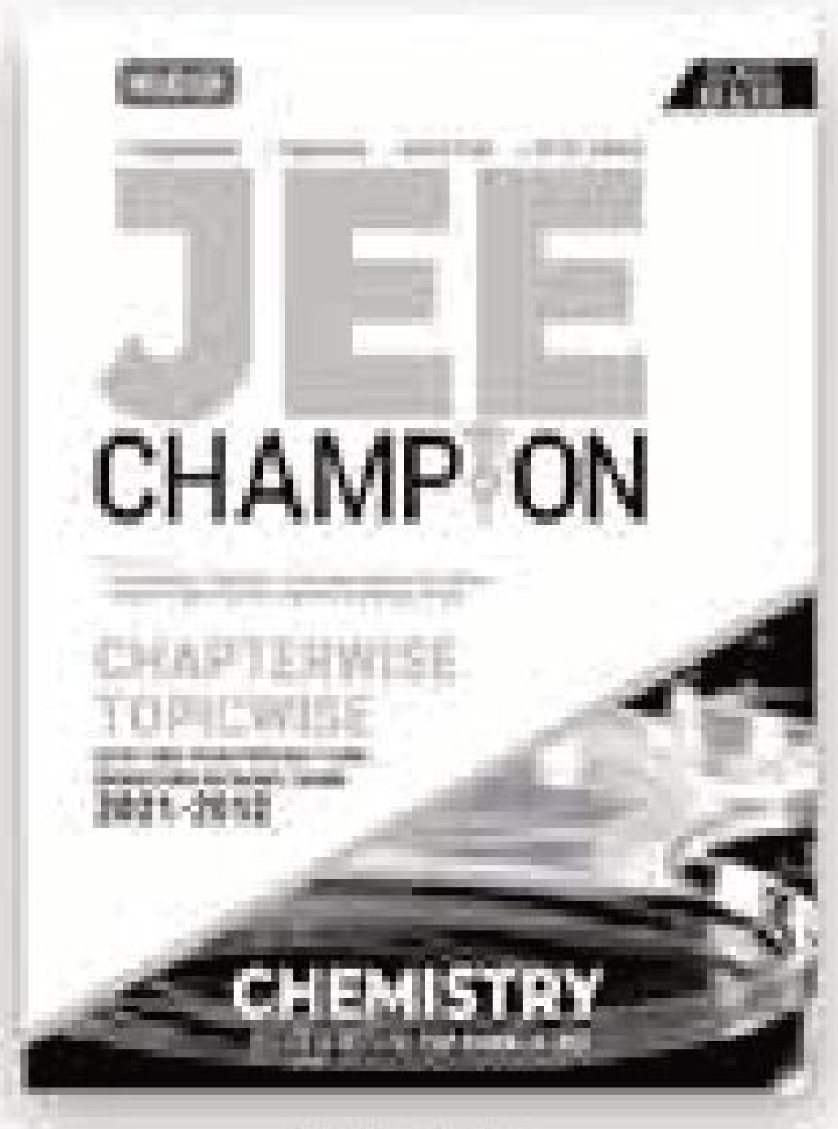
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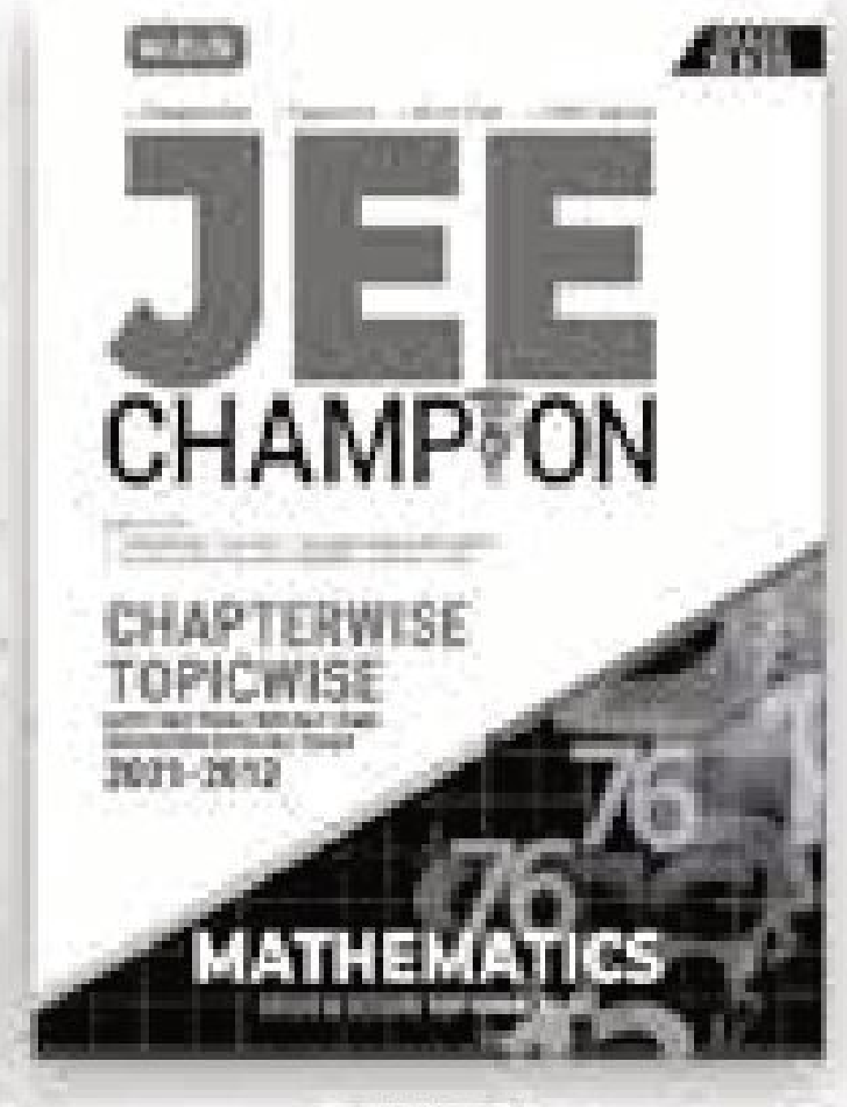
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MATHEMATICS today

Vol. XL No. 1 January 2022

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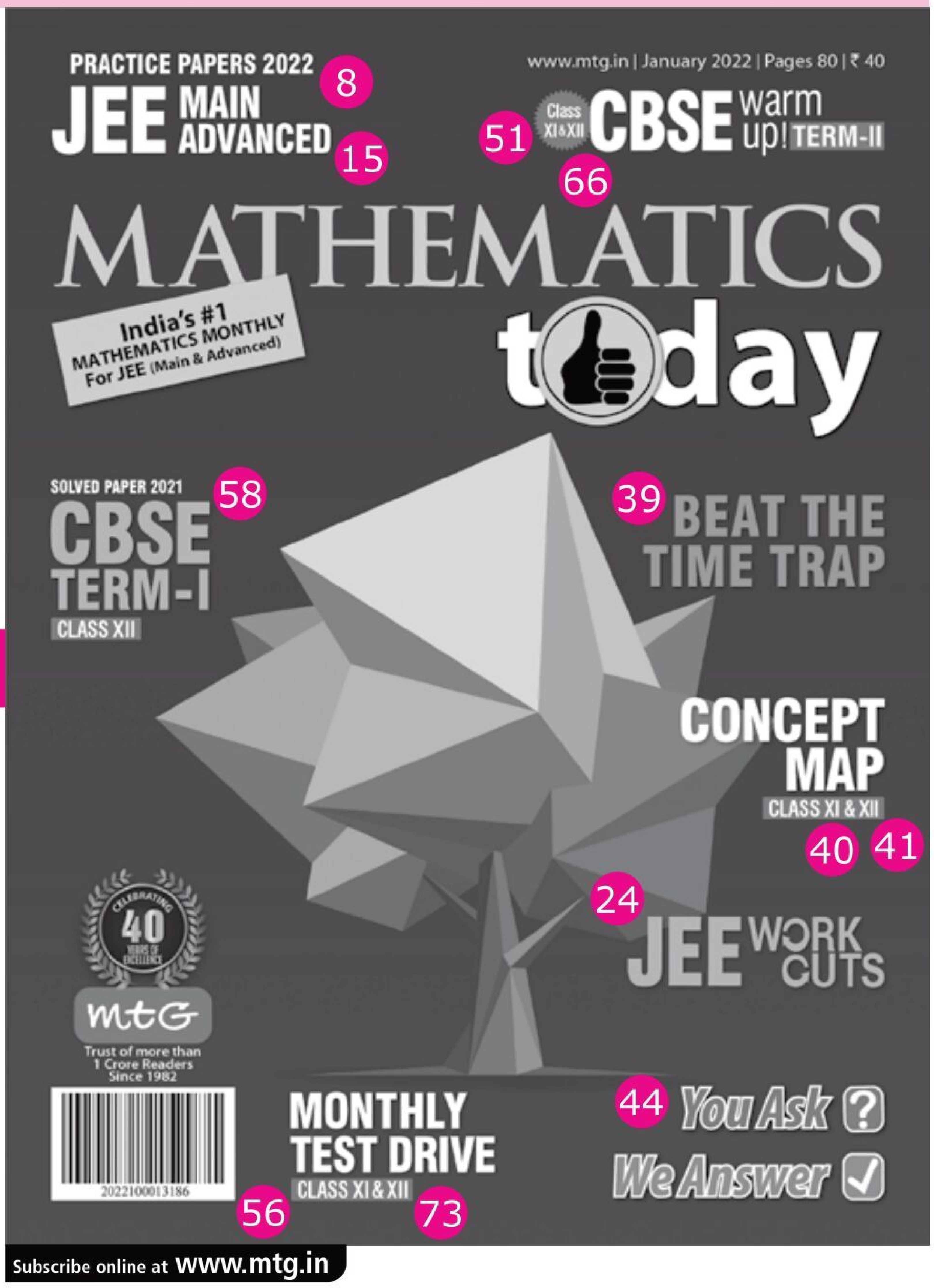

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Printed and Published by Mahabir Singh on behalf of MTG Learning Media Pvt. Ltd. Printed at HT Media Ltd., B-2, Sector-63, Noida, UP-201307 and published at 406, Taj Apartment, Ring Road, Near Safdarjung Hospital, New Delhi - 110029.
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PRACTICE PAPER 2022

JEE MAIN

Time : 1 hr. **Series - 4**

The entire syllabus of Mathematics of JEE MAIN is being divided into four parts. Each part contains 4 units. There will be a Practice Paper which will be published in the subsequent issues. The syllabus for module break-up is given below:

Units	Topic	Syllabus In Detail
XIII	Vector Algebra	Vectors and scalars, the addition of vectors, components of a vector in two dimensions and three-dimensional space, scalar and vector products, scalar and vector triple product.
XIV	Statistics and Probability	Measures of discretion; calculation of mean, median, mode of grouped and ungrouped data calculation of standard deviation, variance and mean deviation for grouped and ungrouped data. Probability: Probability of an event, addition and multiplication theorems of probability, Baye's theorem, probability distribution of a random variate, Bernoulli trials and binomial distribution.
XV	Trigonometry	Trigonometrical identities and equations, trigonometrical functions, inverse trigonometrical functions and their properties, heights and distances.
XVI	Mathematical Reasoning	Statement logical operations and, or, implies, implied by, if and only if, understanding of tautology, contradiction, converse and contrapositive.

SINGLE OPTION CORRECT TYPE

1. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are reciprocal vectors then

$(l\vec{a}+m\vec{b}+n\vec{c})\cdot(l\vec{p}+m\vec{q}+n\vec{r})=$

(a) $l+m+n$ (b) $l^3+m^3+n^3$

(c) $l^2+m^2+n^2$ (d) None of these

2. The expression cos A cos $2A$ cos $4A$ cos $8A$ ... cos $2^{n-1}A$ equals

(a) $\dfrac{2^n \sin A}{\sin 2^n A}$ (b) $\dfrac{\sin 2^n A}{2^n \sin A}$

(c) $\dfrac{2^n \sin 2^n A}{\sin A}$ (d) $\dfrac{\sin A}{2^n \sin 2^n A}$

3. Two squares are chosen from a chess board. The probability that they are of different colour, is

(a) $\dfrac{63}{64}$ (b) $\dfrac{32}{63}$

(c) $\dfrac{23}{64}$ (d) None of these

4. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4 respectively. The variance of the combined data is

(a) $\dfrac{11}{2}$ (b) 6

(c) $\dfrac{13}{2}$ (d) $\dfrac{5}{2}$

5. Which of the following is not true ?

(a) $\{(p \to q) \wedge (q \to r)\} \to (p \to r)$ is a tautology

(b) $(p \wedge \sim q) \leftrightarrow (p \to q)$ is a tautology

(c) $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$

(d) $p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$

6. The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled, then $\vec{R}$ is doubled. If the direction of $\vec{Q}$ is reversed, then $\vec{R}$ is again doubled. Then $P^2 : Q^2 : R^2 =$

(a) 3 : 1 : 1 (b) 2 : 3 : 2

(c) 1 : 2 : 3 (d) 2 : 3 : 1

7. In an experiments with 15 observation on x, the following results were available.

$$\Sigma x^2 = 2830,\ \Sigma x = 170$$

One observation that was 20 was found to be wrong and was replaced by the correct value 30. Then the corrected variance is

(a) 78 (b) 188.66
(c) 177.33 (d) 8.33

8. The number of ways of distributing 20 fruits among 5 people, so that no one receives less than 3 fruits, is

(a) 126 (b) 216 (c) 196 (d) 304

9. If $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(a) 4 (b) $2\sin 2\alpha$
(c) $-4\sin^2\alpha$ (d) $4\sin^2\alpha$

10. Consider the following two statements :

P : If 7 is an odd number, then 7 is divisible by 2.

Q : If 7 is a prime number, then 7 is an odd number.

If V_1 is the truth value of the contrapositive of P and V_2 is the truth value of contrapositive of Q, then the ordered pair (V_1, V_2) equals

(a) (F, F) (b) (F, T) (c) (T, F) (d) (T, T)

11. If $\hat{n}_1, \hat{n}_2$ are two unit vectors and θ is the angle between them, then $\cos\theta/2 =$

(a) $\frac{1}{2}|\hat{n}_1 - \hat{n}_2|$ (b) $\frac{1}{2}(\hat{n}_1 \cdot \hat{n}_2)$

(c) $\frac{1}{2}(|\hat{n}_1 + \hat{n}_2|)$ (d) $\frac{1}{2}(|\hat{n}_1 \times \hat{n}_2|)$

12. Coefficient of variations of two distributions are 55 and 65, and their standard deviations are 22 and 39 respectively. Their arithmetic means are respectively

(a) 15, 20 (b) 40, 60
(c) 30, 50 (d) none of these

13. The probability that a teacher will give a unannounced test is $\frac{1}{4}$. If a student was absent twice, then probability that he will missed at least one test, is

(a) $\frac{1}{2}$ (b) $\frac{7}{16}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

14. The function $f:\left[-\frac{1}{2}, \frac{1}{2}\right] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ defined by $\sin^{-1}(3x - 4x^3)$ is

(a) both one-one onto
(b) onto but not one-one
(c) one-one but not onto
(d) neither one-one nor onto

15. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to

(a) $s \vee (r \vee \sim s)$ (b) $s \wedge r$
(c) $s \wedge \sim r$ (d) $s \wedge (r \wedge \sim s)$

16. If $\vec{r}\cdot\vec{a} = \vec{r}\cdot\vec{b} = \vec{r}\cdot\vec{c} = \frac{1}{2}$ for some non-zero vector $\vec{r}$, then the area of the Δ formed by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is

(a) $|[\vec{a}\ \vec{b}\ \vec{c}]|$ (b) $|\vec{r}|$
(c) $|[\vec{a}\ \vec{b}\ \vec{c}]\vec{r}|$ (d) None of these

17. The mean of the values 0, 1, 2, 3, ..., n with the corresponding weights ${}^nC_0, {}^nC_1, \ldots, {}^nC_n$ respectively, is

(a) $\frac{n+1}{2}$ (b) $\frac{n}{2}$ (c) $\frac{2^n - 1}{2}$ (d) $\frac{2^n + 1}{2}$

18. How many times a person toss a fair coin so that the probability of having atleast one tail is more than 80%?

(a) At least 3 times (b) At most 3 times
(c) at least 5 times (d) at least 6 times

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19. If α, β are solutions of $\sin^2 x + a\sin x + b = 0$ and $\cos^2 x + c\cos x + d = 0$, then $\sin(\alpha+\beta)$ equals

(a) $\dfrac{2ac}{a^2+c^2}$ (b) $\dfrac{a^2+c^2}{2ac}$

(c) $\dfrac{2bd}{b^2+d^2}$ (d) $\dfrac{b^2+d^2}{2bd}$

20. The proposition $\sim(p \Rightarrow q) \Rightarrow (\sim p \vee \sim q)$ is

(a) a tautology (b) a contradiction

(c) either (a) or (b) (d) neither (a) nor (b)

NUMERICAL ANSWER TYPE

21. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be vectors with magnitudes 3, 4 and 5 respectively and $\vec{a}+\vec{b}+\vec{c}=0$, then value of $|\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}|$ is ______.

22. If $x_1, x_2, ..., x_{18}$ are observations such that $\sum_{j=1}^{18}(x_j-8)=9$ and $\sum_{j=1}^{18}(x_j-8)^2=45$, then the standard deviation of these observations is ______.

23. Each of m urns consisting 6 red and 8 black balls. The $(m+1)^{\text{th}}$ urn consisting 7 red and 7 black balls. One of the $(m+1)$ urns is selected randomly and two balls are drawn from it without replacement and found to be black. If the probability that $(m+1)^{\text{th}}$ urn was selected to draw the ball is $\dfrac{1}{17}$, then the value of m equal ______.

24. The value of $3\tan^6 10° - 27\tan^4 10° + 33\tan^2 10°$ equals ______.

25. The volume of the parallelopiped whose edges are $\overrightarrow{OA} = 2\hat{i} - 3\hat{j}$, $\overrightarrow{OB} = \hat{i} + \hat{j} - \hat{k}$, $\overrightarrow{OC} = 3\hat{i} - \hat{k}$ is ______.

26. Let $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ be three vectors such that $\vec{\alpha}\cdot(\vec{\beta}+\vec{\gamma})+\vec{\beta}\cdot(\vec{\gamma}+\vec{\alpha})+\vec{\gamma}\cdot(\vec{\alpha}+\vec{\beta})=0$ and $|\vec{\alpha}|=\sqrt{3}, |\vec{\beta}|=2$ and $|\vec{\gamma}|=3$ then $|\vec{\alpha}+\vec{\beta}+\vec{\gamma}|$ is ______.

27. In a triangle ABC, $b = c$ and $A = 106°$. If M is an interior point such that $\angle MBA = 7°$, $\angle MAB = 23°$ and $\angle MCA = \theta$, then the last digit of θ is ________.

28. The sum and product of mean and variance of a binomial distribution are 24 and 128 respectively, then the value of n is ______.

29. The number of solutions of the equation

$$2\sin^{-1}(\sqrt{x^2-x+1})+\cos^{-1}(\sqrt{x^2-x})=\frac{3\pi}{2}$$

is ______.

30. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar and $\vec{a}+\vec{b}+\vec{c}=\alpha\vec{d}$, $\vec{b}+\vec{c}+\vec{d}=\beta\vec{a}$, then $\vec{a}+\vec{b}+\vec{c}+\vec{d}=$ ______.

SOLUTIONS

1. (c) : $\vec{p}, \vec{q}, \vec{r}$ are reciprocal vectors of $\vec{a}, \vec{b}, \vec{c}$ respectively.

$\therefore \quad \vec{p}\cdot\vec{a}=1$ and $\vec{p}\cdot\vec{b}=\vec{p}\cdot\vec{c}=0$ etc.

$\therefore \quad (l\vec{a}+m\vec{b}+n\vec{c})\cdot(l\vec{p}+m\vec{q}+n\vec{r}) = l^2+m^2+n^2$

2. (b) : $\cos 2^0A \cos 2^1A \cos 2^2A \cos 2^3A \ldots \cos 2^{n-1}A$

$= \dfrac{1}{2\sin A}(\sin 2A \cos 2A)\cos 2^2A \cos 2^3A \ldots \cos 2^{n-1}A$

(∵ $\sin 2A = 2\sin A\cos A$)

$= \dfrac{1}{2^2\sin A}(\sin 4A\cos 4A)\cos 2^3A \ldots\ldots \cos 2^{n-1}A$

$= \dfrac{1}{2^3\sin A}(2\sin 4A\cos 4A)(\cos 2^3A)\ldots \cos 2^{n-1}A$

Repeating the process n times, we get $= \dfrac{\sin 2^nA}{2^n\sin A}$

3. (b) : Number of colours = 2 (Black and white)

Number of squares = 64

Total number of cases (sample space) = ${}^{64}C_2$

Favourable number of cases = ${}^{32}C_1 \times {}^{32}C_1$

$\therefore$ Required probability $= \dfrac{32\times32\times2}{64\times63} = \dfrac{32}{63}$

4. (a) : Data : x_1, x_2, x_3, x_4, x_5

$\bar{x} = \dfrac{\Sigma x_i}{5} = 2$, $\sigma_{x^2} = \dfrac{\Sigma x_i^2}{5} - \bar{x}^2 = 4$

$\therefore \quad \Sigma x_i^2 = 40$

Data : y_1, y_2, y_3, y_4, y_5

$\bar{y} = \dfrac{\Sigma y_i}{5} = 4$, $\sigma_{y^2} = \dfrac{\Sigma y_i^2}{5} - \bar{y}^2 = 5$

$\therefore \; \Sigma y_i^2 = 105 \quad \therefore$ Combined mean $= \dfrac{\Sigma(x_i+y_i)}{10} = 3$

Variance of combined data $= \dfrac{\Sigma x_i^2 + \Sigma y_i^2}{10} - 3^2$

$= \dfrac{145}{10} - 9 = \dfrac{11}{2}$.

5. (b) : Only option (b) is not true as

p	q	$\sim q$	$p \wedge \sim q$	$p \rightarrow q$	$(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$
T	T	F	F	T	F
T	F	T	T	F	F

F	T	F	F	T	F
F	F	T	F	T	F

Hence, $(p \wedge \sim q) \leftrightarrow (p \to q)$ is a fallacy

6. (b): $\vec{P}+\vec{Q}=\vec{R} \Rightarrow P^2+Q^2+2\vec{P}\cdot\vec{Q}=R^2$ ...(i)

$|\vec{P}+2\vec{Q}|=2|\vec{R}| \Rightarrow P^2+4Q^2+4\vec{P}\cdot\vec{Q}=4R^2$ ...(ii)

$|\vec{P}-\vec{Q}|=2|\vec{R}| \Rightarrow P^2+Q^2-2\vec{P}\cdot\vec{Q}=4R^2$ ...(iii)

(i) + (iii) $\Rightarrow 2P^2+2Q^2-5R^2=0$ ...(iv)

(ii) + 2 (iii) $\Rightarrow 3P^2+6Q^2-12R^2=0$

or $P^2+2Q^2-4R^2=0$ ... (v)

From (iv), (v), $\dfrac{P^2}{2}=\dfrac{Q^2}{3}=\dfrac{R^2}{2}$.

7. (a): $n = 15, \Sigma x^2 = 2830, \Sigma x = 170$

(New)$\Sigma x^2 = 2830 - 400 + 900 = 3330$

(New)$\Sigma x = 170 - 20 + 30 = 180$

$\therefore$ Correct Variance,

$$\sigma^2=\frac{\sum x^2}{n}-\left(\frac{\sum x}{n}\right)^2=\frac{3330}{15}-\left(\frac{180}{15}\right)^2=78$$

8. (a): $x_1+x_2+x_3+x_4+x_5=20, X_i=x_i-2,$ $i = 1, 2, .., 5.$

$\Rightarrow X_1+X_2+X_3+X_4+X_5=10, X_i \geq 1.$

The number of positive integer solutions is $\binom{10-1}{5-1}=\binom{9}{4}=126$

9. (d): Using $\cos^{-1}A-\cos^{-1}B$

$$=\cos^{-1}\left(AB+\sqrt{(1-A^2)}\sqrt{(1-B^2)}\right)$$

$$\therefore \cos^{-1}x-\cos^{-1}\frac{y}{2}=\alpha$$

$$\Rightarrow \frac{xy}{2}+\sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}}=\cos\alpha$$

$$\Rightarrow \left(\cos\alpha-\frac{xy}{2}\right)^2=(1-x^2)\left(1-\frac{y^2}{4}\right)$$

$$\Rightarrow \cos^2\alpha+\frac{x^2y^2}{4}-xy\cos\alpha=1+\frac{x^2y^2}{4}-x^2-\frac{y^2}{4}$$

$$\Rightarrow 4x^2-4xy\cos\alpha+y^2=4(1-\cos^2\alpha)=4\sin^2\alpha.$$

10. (b): We have,

Contrapositive of P : If 7 is not divisible by 2, then 7 is not an odd number. T $\Rightarrow$ F : F(V_1)

Contrapositive of Q : If 7 is not an odd number, then 7 is not a prime number. F $\Rightarrow$ F : T (V_2)

11. (c): $(|\hat{n}_1+\hat{n}_2|)^2=|\hat{n}_1|^2+|\hat{n}_2|^2+2|\hat{n}_1||\hat{n}_2|\cos\theta$

$= 2(1+\cos\theta) = 4\cos^2\theta/2$

$$\therefore \cos\frac{\theta}{2}=\frac{1}{2}|\hat{n}_1+\hat{n}_2|$$

12. (b): Coefficient of variation $=\dfrac{\sigma}{\bar{x}}\times 100$

For C.V. = 55, $\sigma = 22$

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$$55=\frac{22}{\bar{x}_1}\times 100 \Rightarrow \bar{x}_1=\frac{22}{55}\times 100=40$$

For C.V. = 65, $\sigma = 39$

$$\bar{x}_2=\frac{39}{65}\times 100=60 \quad \therefore \text{ Means are 40, 60.}$$

13. (b) : The probability that only one test was held in those day in which the student was absent

$$=2\times\frac{1}{4}\times\frac{3}{4}=\frac{3}{8}$$

Now, the probability that the test was held on both the days $=\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$

$\therefore$ The probability that student missed at least one test $=\frac{3}{8}+\frac{1}{16}=\frac{7}{16}$

14. (a) : $f(x) = \sin^{-1}(3x - 4x^3) = 3\sin^{-1}x$

Clearly, $3\sin^{-1}x \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ if $\sin^{-1}x \in \left[-\frac{\pi}{6},\frac{\pi}{6}\right]$

if $x \in \left[-\sin\frac{\pi}{6},\sin\frac{\pi}{6}\right]$ or if $x \in \left[-\frac{1}{2},\frac{1}{2}\right]$,

which is obvious

$\therefore$ f is onto.

Now, say $x_1, x_2 \in \left[-\frac{1}{2},\frac{1}{2}\right]$ such that

$f(x_1) = f(x_2) \Rightarrow 3x_1 - 4x_1^3 = 3x_2 - 4x_2^3$

$\Rightarrow x_1 - x_2 = 0 \;\therefore\; x_1 = x_2$

So, f is one-one also.

15. (b): Using the rules of logic, we have $\sim s \vee (\sim r \wedge s)$ $= (\sim s \vee \sim r) \wedge (\sim s \vee s) = \sim s \vee \sim r$

Now the negation of above is $\sim(\sim s \vee \sim r) = s \wedge r$

16. (c) : Let $\vec{r} = x(\vec{a}\times\vec{b}) + y(\vec{b}\times\vec{c}) + z(\vec{c}\times\vec{a})$

Taking dot product with $\vec{a}, \vec{b}$ and $\vec{c}$, we get

$$x=\frac{\vec{r}\cdot\vec{c}}{[\vec{a}\;\vec{b}\;\vec{c}]},\; y=\frac{\vec{r}\cdot\vec{a}}{[\vec{a}\;\vec{b}\;\vec{c}]},\; z=\frac{\vec{r}\cdot\vec{b}}{[\vec{a}\;\vec{b}\;\vec{c}]}$$

$$\therefore\; [\vec{a}\,\vec{b}\,\vec{c}]\,\vec{r}=\frac{1}{2}(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})$$

$$\therefore\; \text{Area of } \Delta ABC=\frac{1}{2}|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|=|[\vec{a}\,\vec{b}\,\vec{c}]\,\vec{r}\,|$$

17. (b): The required mean is

$$\bar{x}=\frac{0\cdot 1+1\cdot {}^nC_1+2\cdot {}^nC_2+3\cdot {}^nC_3+\ldots+n\cdot {}^nC_n}{1+{}^nC_1+{}^nC_2+\ldots+{}^nC_n}$$

$$=\frac{\sum_{r=0}^{n} r\cdot {}^nC_r}{\sum_{r=0}^{n} {}^nC_r}=\frac{\sum_{r=1}^{n} r\cdot\frac{n}{r}\,{}^{n-1}C_{r-1}}{\sum_{r=0}^{n} {}^nC_r}=\frac{n\sum_{r=1}^{n} {}^{n-1}C_{r-1}}{\sum_{r=0}^{n} {}^nC_r}=\frac{n\cdot 2^{n-1}}{2^n}=\frac{n}{2}$$

18. (a): Let the person tosses the coin n times, X denotes the number of tails.

$\therefore$ $P(X \geq 1) > 80\%$

$$\Rightarrow\; 1-P(X=0)>\frac{80}{100} \Rightarrow 1-{}^nC_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^n>\frac{4}{5}$$

$$\left(\because\; P(X=r)={}^nC_r\,p^{n-r}q^r={}^nC_r\left(\frac{1}{2}\right)^n\right)$$

$$\Rightarrow\; 1-\frac{1}{2^n}>\frac{4}{5} \Rightarrow \frac{1}{2^n}<\frac{1}{5}$$

$\Rightarrow$ $2^n > 5$, which can be true only if least value of $n = 3$.

Therefore, coin must be tossed at least 3 times.

19. (a): Given, α, β are roots of the equations $\sin^2 x + a\sin x + b = 0$ and $\cos^2 x + c\cos x + d = 0$

$\therefore$ $\sin^2\alpha + a\sin\alpha + b = 0$ and $\sin^2\beta + a\sin\beta + b = 0$

$\Rightarrow$ $\sin\alpha + \sin\beta = -a$ ...(A)

Similarly, $\cos^2\alpha + c\cos\alpha + d = 0$ and $\cos^2\beta + c\cos\beta + d = 0$

$\Rightarrow$ $\cos\alpha + \cos\beta = -c$ ...(B)

Now from (A), we have

$$\left.\begin{array}{l} 2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}=-a \\ \text{and from (B), we have} \\ 2\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}=-c \end{array}\right\} \quad \text{...(C)}$$

On dividing the equations of (C), we have $\tan\frac{\alpha+\beta}{2}=\frac{a}{c}$

$$\therefore\; \sin(\alpha+\beta)=\frac{2\tan\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)}=\frac{2ac}{a^2+c^2}$$

20. (a) : By truth table

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$\sim(p \Rightarrow q)$	$\sim p \vee \sim q$	$\sim(p \Rightarrow q) \Rightarrow (\sim p \vee \sim q)$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	T	T

Hence, given proposition is a tautology.

21. (25) : $\vec{a}, \vec{b}$ and $\vec{c}$ form the right angled triangle as $c^2 = a^2 + b^2$

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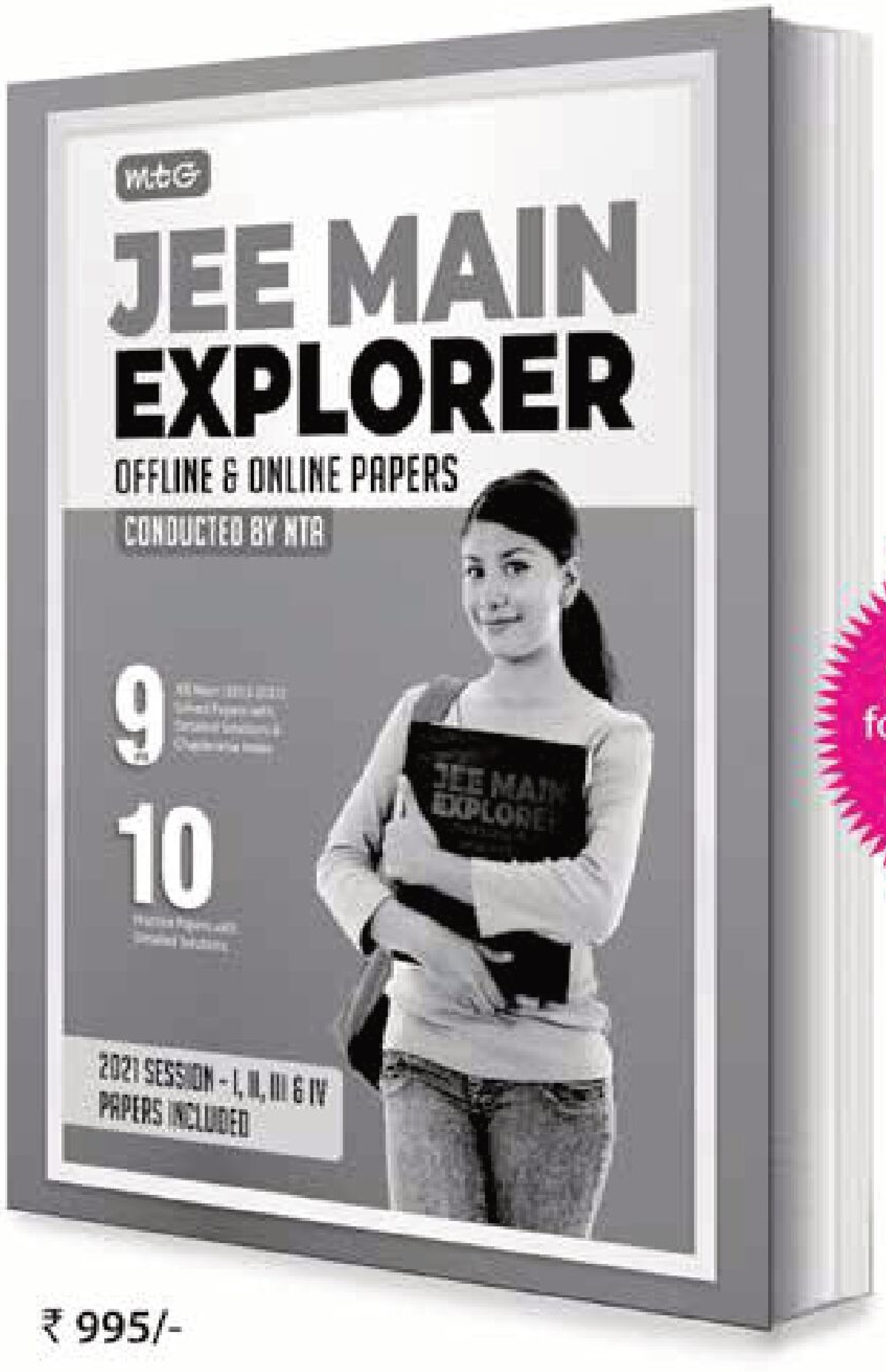
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∴ $\vec{a}\cdot\vec{b}=0$

⇒ $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\vec{c}\cdot(\vec{a}+\vec{b})=\vec{c}\cdot(-\vec{c})$ (∵ $\vec{a}+\vec{b}+\vec{c}=\vec{0}$)

$= -|\vec{c}|^2 = -25$

22. (1.5) : $\sum_{j=1}^{18}(x_j-8)=9 \Rightarrow \sum_{j=1}^{18}x_j=153$

Also, $\sum_{j=1}^{18}(x_j-8)^2=45$

$$\Rightarrow \sum_{j=1}^{18}x_j^2=45-64\times18+16\times153=1341$$

So, standard deviation $=\sqrt{\dfrac{\Sigma x_j^2}{n}-\left(\dfrac{\Sigma x_j}{n}\right)^2}$

$$=\sqrt{\frac{1341}{18}-\left(\frac{153}{18}\right)^2}=\sqrt{74.5-72.25}=1.5$$

23. (12) : Given, each m urns having 6 red and 8 black balls and $(m+1)^{\text{th}}$ urn consists 7 red and 7 black balls. Let A_1 and A_2 denotes the events that one of first m urns and $(m+1)^{\text{th}}$ urn is selected respectively.

∴ $P(A_1)=\dfrac{m}{m+1}$, $P(A_2)=\dfrac{1}{m+1}$

Let A be the event that two balls drawn without replacement from the urn are black.

$$\therefore\ P(A\,|\,A_1)=\frac{{}^8C_2}{{}^{14}C_2}=\frac{8}{14}\times\frac{7}{13}=\frac{4}{13}$$

$$\text{and } P(A\,|\,A_2)=\frac{{}^7C_2}{{}^{14}C_2}=\frac{7}{14}\times\frac{6}{13}=\frac{3}{13}$$

$$\therefore\ P(A)=P(A_1)P(A\,|\,A_1)+P(A_2)P(A\,|\,A_2)$$

$$\therefore\ P(A)=\left(\frac{m}{m+1}\right)\left(\frac{4}{13}\right)+\frac{1}{(m+1)}\left(\frac{3}{13}\right)=\frac{4m+3}{13(m+1)}$$

Using Baye's theorem, we have

$$P(A_2\,|\,A)=\frac{P(A_2)P(A\,|\,A_2)}{P(A)}=\frac{\left(\frac{1}{m+1}\right)\left(\frac{3}{13}\right)}{\frac{4m+3}{13(m+1)}}$$

∴ $\dfrac{1}{17}=\dfrac{3}{4m+3}$ ⇒ $4m+3=51$ ⇒ $m=12$.

24. (1) : Using fact : $\tan 3A=\dfrac{3\tan A-\tan^3 A}{1-3\tan^2 A}$

Putting $A=10°$

∴ $\dfrac{1}{\sqrt{3}}(1-3\tan^2 10°)=\tan 10°(3-\tan^2 10°)$

⇒ $(1-3\tan^2 10°)^2=3\tan^2 10°(3-\tan^2 10°)^2$

⇒ $3\tan^2 10°(9+\tan^4 10°-6\tan^2 10°)$
$=1+9\tan^4 10°-6\tan^2 10°$

⇒ $3\tan^6 10°-27\tan^4 10°+33\tan^2 10°=1$

25. (4) : Volume is

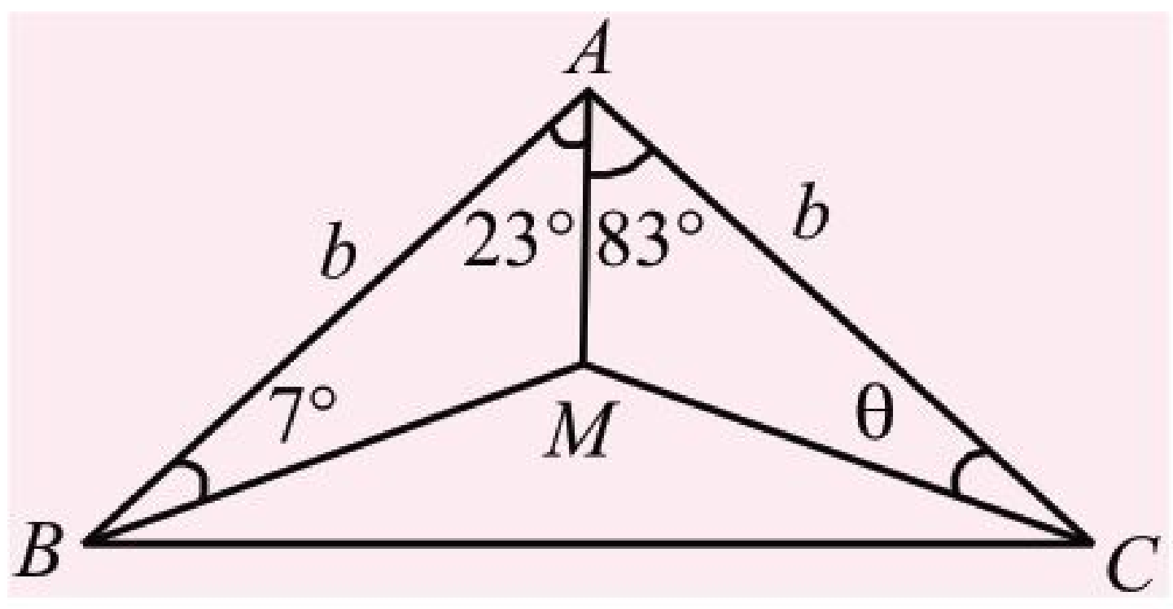
$$\left[\overrightarrow{OA}\ \overrightarrow{OB}\ \overrightarrow{OC}\right]=\begin{vmatrix}2 & -3 & 0\\ 1 & 1 & -1\\ 3 & 0 & -1\end{vmatrix}=-2+6=4$$

26. (4) : Since, $|\vec{\alpha}+\vec{\beta}+\vec{\gamma}|^2=(\vec{\alpha}+\vec{\beta}+\vec{\gamma})\cdot(\vec{\alpha}+\vec{\beta}+\vec{\gamma})$

$=|\vec{\alpha}|^2+|\vec{\beta}|^2+|\vec{\gamma}|^2+\Sigma\vec{\alpha}\cdot(\vec{\beta}+\vec{\gamma})=3+4+9+0=16$

⇒ $|\vec{\alpha}+\vec{\beta}+\vec{\gamma}|=4$

27. (4) : By sine rule,



$\dfrac{AM}{\sin 7°}=\dfrac{b}{\sin 30°}$,

$\dfrac{AM}{\sin\theta}=\dfrac{b}{\sin(\theta+83°)}$

∴ $2\sin 7°\times\sin(\theta+83°)=\sin\theta$

⇒ $\cos(\theta+76°)-\cos(\theta+90°)=\sin\theta$

⇒ $\cos(\theta+76°)=0 \Rightarrow \theta=14°$

28. (32) : Given, $np+npq=24$ and $np\,(npq)=128$

⇒ $np\,(1+q)=24$ and $n^2p^2q=128$

∴ $\dfrac{(1+q)^2}{q}=\dfrac{24\times24}{128}$

⇒ $2q^2-5q+2=0$ ⇒ $p=q=1/2$

Now, $np\,(1+q)=24$

∴ $n\left(\dfrac{1}{2}\right)\left(\dfrac{3}{2}\right)=24 \Rightarrow n=\dfrac{24\times4}{3} \Rightarrow n=32$

29. (2) : $\sin^{-1}\sqrt{x}, \cos^{-1}\sqrt{x}$ are defined for $0\le x\le 1$

∴ $x^2-x+1\le 1$ and $x^2-x\ge 0$

⇒ $x^2-x\le 0$ and $x^2-x\ge 0$

⇒ $x^2-x=0$ ⇒ $x=1, 0$

∴ There are two solutions, both satisfies the equation.

30. (0): $\vec{a}+\vec{b}+\vec{c}=\alpha\vec{d}$, $\vec{b}+\vec{c}+\vec{d}=\beta\vec{a}$

Eliminating $\vec{d}$, $\vec{a}+\vec{b}+\vec{c}=\alpha(\beta\vec{a}-\vec{b}-\vec{c})$

∴ $(1-\alpha\beta)\vec{a}+(1+\alpha)\vec{b}+(1+\alpha)\vec{c}=0$

Since $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar

⇒ $1-\alpha\beta=0,\ 1+\alpha=0$ ⇒ $\alpha=-1, \beta=-1$

$\vec{a}+\vec{b}+\vec{c}=\alpha\vec{d}=-\vec{d}$

∴ $\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$.

◆◆

JEE 2022
PRACTICE PAPER
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Topics to be covered : Matrices, Trigonometry and Probability

PAPER-1

SECTION 1

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options. (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*

1. Two numbers are selected at random from a set of first 120 natural numbers. Two numbers are selected randomly, the probability that product of selected number is divisible by 3.

(a) $\frac{13}{357}$ (b) $\frac{199}{357}$ (c) $\frac{158}{357}$ (d) $\frac{160}{357}$

2. The complex numbers $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate of each other for $(n \in I)$ $x =$

(a) $n\pi$ (b) 0

(c) $(2n+1)\frac{\pi}{2}$ (d) no value of x

3. Let three matrices $A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$, then $tr(A) + tr\left(\frac{ABC}{2}\right) + tr\left(\frac{A(BC)^2}{4}\right) + tr\left(\frac{A(BC)^3}{8}\right) + ... + \infty =$

(a) 6 (b) 9

(c) 12 (d) none of these

4. A coin is tossed n times. If the probability of getting at least two heads is less than that of getting at least one tail by $\frac{3}{32}$, then the value of n equals

(a) 3 (b) 4 (c) 5 (d) 6

SECTION 2

- *This section contains THREE (03) questions stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 5 and 6

Let 3 × 3 matrices are formed using the elements from the set $\{-3, -2, -1, 0, 1, 2, 3\}$.

5. If number of matrices that are lower triangular or upper triangular is $7^{\lambda}(2 \cdot 7^{\lambda} - 1)$, then λ equals ________.

6. If number of matrices each of whose trace is atleast 7 is $k \cdot 7^6$, then k equals _________.

Question Stem for Question Nos. 7 and 8

Let $a = \cos^{-1}(\cos 20)$, $b = \cos^{-1}(\cos 30)$ and $c = \sin^{-1}(\sin(a + b))$.

7. Maximum value of $\sin(2(a + b + c)x) + \cos^2((a + b + c)x)$ is ________.

8. The largest integer x for which $\sin^{-1}(\sin x) \geq |x - (a + b + c)|$ is _______.

Question Stem for Question Nos. 9 and 10

A bag contains 9 black, 7 white and 4 green balls. 3 balls are drawn randomly.

9. Probability of getting 3 balls of different colours is _______.

10. Probability of getting 2 black and one green balls is _______.

SECTION 3

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***

11. For all x in $(-1, 1)$, let $A(x)$ be a matrix defined by

$$A(x) = \sqrt{1-x^2}\begin{pmatrix} 1 & -x \\ -x & 1 \end{pmatrix},\text{ then}$$

(a) $A(x)\,A(y) = A\,(xy)$

(b) $A(x)\,A(y) = A\left(\dfrac{x+y}{1+xy}\right)$

(c) $A(x)\,A(y) = A\left(\dfrac{x-y}{1+xy}\right)$

(d) $(A(x))^{-1} = A(-x)$

12. The expression $(\alpha\tan\gamma + \beta\cot\gamma)(\alpha\cot\gamma + \beta\tan\gamma) - 4\alpha\beta\cot^2 2\gamma$ is

(a) independent of α, β (b) independent of γ

(c) dependent on γ (d) dependent on α, β

13. The independent probabilities that A, B, C can solve the problem are $\dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{4}$ respectively, then

(a) Probability that the problem will be solved is $\dfrac{3}{4}$

(b) Probability that problem can not be solved is $\dfrac{1}{4}$

(c) Probability that only two of them solve the problem is $\dfrac{1}{4}$

(d) Probability exactly one of then solve the problem is $\dfrac{11}{24}$

14. In a triangle if $\Delta = a^2 - (b-c)^2$, then which of the following is/are correct?

(a) $\tan\dfrac{A}{2} = \dfrac{1}{4}$ (b) $\operatorname{cosec}A = \dfrac{17}{8}$

(c) $\cot A = \dfrac{15}{8}$ (d) $\sin\dfrac{A}{2} = \dfrac{1}{\sqrt{17}}$

15. Suppose A and B are two 3×3 invertible matrices such that $(AB)^k = A^kB^k$ for $k = 2008, 2009, 2010$, then

(a) $AB^{-1}A^{-1} = B^{-1}$ (b) $A^{-1}B^{-2}A = B^2$

(c) $AB = BA$ (d) $A^{-2}BA^2 = A^{-1}BA$

16. Three numbers are selected at random without replacement from $\{1, 2, 3, ..., 11\}$. Let A_1 be the event that minimum of selected numbers is 4 and A_2 be the event that their maximum is 7, then

(a) $P(A_1) = \dfrac{7}{55}$ (b) $P(A_2) = \dfrac{1}{11}$

(c) $P\left(\dfrac{A_2}{A_1}\right) = \dfrac{2}{21}$ (d) $P\left(\dfrac{A_1}{A_2}\right) = \dfrac{2}{15}$

SECTION 4

- ***This section contains THREE (03) questions.***
- ***The answer to each question is a NON-NEGATIVE INTEGER.***

17. If $(\sin^{-1}x)^2 + (\sin^{-1}y)^2 + (\sin^{-1}z)^2 = \dfrac{3\pi^2}{4}$, then the maximum value of $(x - y + z)$ is ________.

18. The papers of 5 students can be checked by any one of the 5 teachers. If the probability that all the 5 papers are checked by exactly 2 teachers is m, then the value of $\dfrac{125m}{2}$ equals ________.

19. Suppose a matrix A satisfies $A^2 - 5A + 7I = O$. If $A^5 = aA + bI$, then the value of $a + \dfrac{3b}{7} + 21$ must be ______.

PAPER-2

SECTION 1

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***

1. If $\theta = \tan^{-1}(2\tan^2\theta) - \tan^{-1}((1/3)\tan\theta)$, then

(a) $\tan\theta = -2$ (b) $\tan\theta = 0$

(c) $\tan\theta = 1$ (d) $\tan\theta = 2$

2. If A is symmetric and B is skew symmetric matrix, then which of the following is/are not true?

(a) ABA^T is symmetric matrix

(b) $AB^T + BA^T$ is symmetric matrix

(c) $(A + B)(A - B)$ is skew symmetric

(d) $(A + I)(B - I)$ is skew symmetric

3. For two events A and B, the probability that exactly one of them occurs is given by

(a) $P(A) + P(B) - 2P(A \cap B)$

(b) $P(A \cup B) - P(A \cap B)$

(c) $P(A') + P(B') - 2P(A' \cap B')$

(d) $P(A' \cap B) + P(A \cap B')$

4. The number of real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ is/are less than

(a) 4 (b) 5 (c) 6 (d) 7

5. Let $C_k = {}^nC_k$ for $0 \le k \le n$ and $A_k = \begin{pmatrix} C_{k-1}^2 & 0 \\ 0 & C_k^2 \end{pmatrix}$ for $k \ge 1$, and $A_1 + A_2 + \dots + A_n = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$, then

(a) $k_1 = k_2$ (b) $k_1 + k_2 = {}^{2n}C_{2n} + 1$

(c) $k_1 = {}^{2n}C_n - 1$ (d) $k_2 = {}^{2n}C_{n+1}$

6. An electric component manufactured by 'RAJU' Electronics is tested for its defectiveness by a sophisticated testing device. Let 'A' denote the event " the device is defective" and 'B' denote the event "the testing device reveals the component to be defective' If $P(A) = \alpha$ and $P\left(\frac{B}{A}\right) = P\left(\frac{B'}{A'}\right) = 1 - \alpha$ where $0 < \alpha < 1$ then which of the following is true?

(a) $P(B) = 2\alpha(1 - \alpha)$ (b) $P(A'/B) = 1/2$

(c) $P(B') = \alpha^2 + (1 - \alpha)^2$

(d) $P\left(\frac{A'}{B'}\right) = \left(\frac{\alpha}{1-\alpha}\right)^2$

SECTION 2

- ***This section contains THREE (03) question stems.***
- ***There are TWO (02) questions corresponding to each question stem.***
- ***The answer to each question is a NUMERICAL VALUE.***

Question Stem for Question Nos. 7 and 8

Let $f(x) = [\tan^{-1}x] - [\cot^{-1}x]$, $x \in [0, \infty)$.

7. If the least value of $f(x)$ is $-\lambda/5$, then λ equals __________.

8. The greatest value of $f(x)$ is __________.

Question Stem for Question Nos. 9 and 10

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^n = \left[a_{ij}\right]^n$, where a_{ij} is any element of i^{th} row and j^{th} column of the matrix A.

9. The value of $\lim\limits_{n\to\infty} \frac{a_{12}}{\left(\sqrt{2}+1\right)^n}$ is __________.

10. The value of $\lim\limits_{n\to\infty} \frac{a_{22}}{(\sqrt{2}+1)^n}$ is __________.

Question Stem for Question Nos. 11 and 12

The contents of urns I, II, III are as follows :

Urn I : 1 White, 2 Black and 3 Red balls
Urn II : 2 White, 1 Black and 1 Red ball
Urn III : 4 White, 5 Black and 3 Red balls

One urn is chosen randomly and two balls are drawn. If E_1, E_2 and E_3 denote the event that urn I, II and III are chosen respectively and E be the event of selection of two balls from the selecting urn as white and red.

11. $P\left(\frac{E_1}{E}\right) =$ _______.

12. $P\left(\frac{E_2}{E}\right) + P\left(\frac{E_3}{E}\right) =$ _______.

SECTION 3

- ***This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO(02) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.***
- ***For each question, choose the option corresponding to the correct answer.***

Paragraph for Q. 13-14

For $x \in \left(0, \frac{\pi}{4}\right)$,

let $S_n = \sum\limits_{r=1}^{2n} \sin(\sin^{-1} x^{3r-2}), C_n = \sum\limits_{r=1}^{2n} \cos(\cos^{-1} x^{3r-1})$

and $T_n = \sum\limits_{r=1}^{2n} \tan(\tan^{-1} x^{3r})$, where $n \in N$ and $n \ge 3$

13. The correct order of S_n, C_n and T_n is given by

(a) $S_n > C_n > T_n$ (b) $C_n > S_n > T_n$

(c) $S_n > T_n > C_n$ (d) $T_n > C_n > S_n$

14. The value of $\lim\limits_{n\to\infty}(S_n + C_n + T_n)$ is equal to

(a) $\frac{1-x}{x}$ (b) $\frac{x}{1-x}$ (c) $\frac{x}{1+x}$ (d) $\frac{1+x}{x}$

Paragraph for Q. 15-16

If A is a symmetric matrix, B is a skew symmetric matrix, $(A + B)^{-1}$ and $(A - B)^{-1}$ exist and $C = (A + B)^{-1}(A - B)$, then

15. $C^T(A + B)C =$

(a) $A + B$ (b) $A - B$ (c) $B - A$ (d) C

16. $C^T(A - B)\,C =$

(a) $A + B$ (b) $A - B$ (c) C (d) $B - A$

SECTION 4

- ***This section contains THREE (03) questions.***
- ***The answer to each question is a NON-NEGATIVE INTEGER.***

17. If A and B be any two events such that $P(A) = 2/5$ and $P(A \cap B) = \frac{3}{20}$ and conditional probability $P(A|A' \cup B') = \frac{l}{m}$, then l equals ________.

18. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then value of $\sum \frac{(x^{101}+y^{101})(x^{202}+y^{202})}{(x^{303}+y^{303})(x^{404}+y^{404})} =$ ________.

19. Let A be a 2×2 matrix with real entries and $\det(A) = t \ne 0$ such that $\det(A + t(\text{adj } A)) = 0$, then $\det(A - t(\text{adj } A)) =$ ________.

SOLUTIONS

PAPER-1

1. (b) : Given set of numbers is $\{1, 2, 3, 4, ..., 120\}$
Now, $\{1, 2, 3, 4, 5, ..., 120\}$

$$=\underbrace{\{1,2,4,5,....,119\}}_{80 \text{ numbers}} \cup \underbrace{\{3,6,9,....,120\}}_{40 \text{ numbers}}$$

The product of two numbers will be divisible by 3 if both the numbers are from the set $\{3, 6, 9, ..., 120\}$ (*i.e.* from 40 numbers) or one from the set $\{1, 2, 4, 5, .., 119\}$ and other from the set $\{3, 6, 9, ..., 120\}$.

Let A = event that product of number is divisible by 3

$\therefore \quad n(A) = {}^{40}C_2 + {}^{80}C_1 \times {}^{40}C_1$ and $n(S) = {}^{120}C_2$

$$\therefore \ P(A)=\frac{{}^{40}C_2+{}^{80}C_1\times{}^{40}C_1}{{}^{120}C_2}=\frac{199}{357}$$

2. (d) : $\sin x - i\cos 2x = \cos x - i\sin 2x$

$\Rightarrow \ \sin x = \cos x$ and $\cos 2x = \sin 2x$,

which is not true for any value of x.

3. (a) : $BC=\begin{bmatrix}3 & 4\\ 2 & 3\end{bmatrix}\begin{bmatrix}3 & -4\\ -2 & 3\end{bmatrix} \Rightarrow BC=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I_2$

$\therefore$ Given expression $= tr(A)+tr\left(\frac{A}{2}\right)+tr\left(\frac{A}{2^2}\right)+ \ ...+\infty$

$$=tr(A)+\frac{1}{2}tr(A)+\frac{1}{2^2}tr(A)+...+\infty=\frac{tr(A)}{1-(1/2)}$$

$= 2tr(A) = 2(2 + 1) = 6$

4. (d) : We have, $P(X = r) = {}^nC_r\, p^r\, q^{n-r}$, $p = q = 1/2$

$\therefore \quad$ Probability of getting at least 2 heads

$$= 1 - P(0) - P(1) = 1-\frac{1}{2^n}-n\cdot\frac{1}{2^n}$$

Probability of getting at least 1 tail

$$= 1 - P(0) = 1-\frac{1}{2^n}$$

Now, as per question we have

$$\left(1-\frac{1}{2^n}\right)-\left(1-\frac{1}{2^n}-\frac{n}{2^n}\right)=\frac{3}{32}$$

$$\Rightarrow \ \frac{n}{2^n}=\frac{3}{32}=\frac{6}{64}=\frac{6}{2^6} \ \Rightarrow \ n=6$$

5. (3) : A = lower triangular matrix
B = upper triangular matrix

$n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$= 7^6 + 7^6 - 7^3 = 2 \cdot 7^6 - 7^3 = 7^3(2 \cdot 7^3 - 1)$

6. (10) :

Trace	Trace elements
9	3,3,3
8	3,3,2
7	3,2,2 (or) 3,3,1

Required number $= 1 \cdot 7^6 + 3(7^6) + 6(7^6) = 10 \cdot 7^6$

7. (1.62) : $a = 20 - 6\pi$, $b = 10\pi - 30$ and
$c = \sin^{-1}\sin(4\pi - 10) = 10 - 3\pi$
So $a + b + c = \pi$
Let $f(x) = \sin(2(a + b + c)x) + \cos^2((a + b + c)x)$

$$= \sin 2\pi x + \cos^2\pi x = \sin 2\pi x+\frac{1+\cos 2\pi x}{2}$$

$$\Rightarrow \ \text{maximum value of } f(x)=\frac{\sqrt{5}+1}{2}=1.62$$

8. (3) : $\sin^{-1}\sin x \ge |x-\pi| \ \Rightarrow \ x\in\left[\frac{\pi}{2},\pi\right]$

So largest integer is $x = 3$.

9. (0.22) : Total balls in the bag = 9 + 7 + 4 = 20

P(balls are of different colour)

$$=\frac{{}^9C_1\times{}^7C_1\times{}^4C_1}{{}^{20}C_3}=\frac{9\cdot 7\cdot 4\cdot 6}{20\cdot 19\cdot 18}=0.22$$

10. (0.126) : P(two black and one green ball)

$$=\frac{{}^9C_2\times{}^4C_1}{{}^{20}C_3}=0.126$$

11. (b, d) : $A(x)\,A(y)$

$$=\sqrt{1-x^2}\sqrt{1-y^2}\begin{pmatrix}1 & -x\\ -x & 1\end{pmatrix}\begin{pmatrix}1 & -y\\ -y & 1\end{pmatrix}$$

$$=\sqrt{1-x^2}\sqrt{1-y^2}\begin{pmatrix}1+xy & -(x+y)\\ -(x+y) & 1+xy\end{pmatrix}$$

$$=\frac{\sqrt{1-x^2}\sqrt{1-y^2}}{1+xy}\begin{pmatrix}1 & -\frac{x+y}{1+xy}\\ -\frac{x+y}{1+xy} & 1\end{pmatrix}$$

$$=\sqrt{1-\left(\frac{x+y}{1+xy}\right)^2}\begin{pmatrix}1 & -\frac{x+y}{1+xy}\\ -\frac{x+y}{1+xy} & 1\end{pmatrix} = A\left(\frac{x+y}{1+xy}\right)$$

Since $A(0) = I$,
putting $y = -x$ in correct choice (b), we get
$A(x)\,A(-x) = A(0) = I \Rightarrow A(-x) = (A(x))^{-1}$

12. (b, d) : We have,
$(\alpha\tan\gamma + \beta\cot\gamma)(\alpha\cot\gamma + \beta\tan\gamma) - 4\alpha\beta\cot^2 2\gamma$

$= \alpha^2 + (\tan^2\gamma + \cot^2\gamma)\,\alpha\beta + \beta^2 - 4\alpha\beta\dfrac{\cos^2 2\gamma}{\sin^2 2\gamma}$

$=\alpha^2+\beta^2+\alpha\beta\left[\left(\dfrac{\sin^2\gamma}{\cos^2\gamma}+\dfrac{\cos^2\gamma}{\sin^2\gamma}\right)-\dfrac{4(\cos^2\gamma-\sin^2\gamma)^2}{4\sin^2\gamma\cos^2\gamma}\right]$

$= \alpha^2 + \beta^2 + \alpha\beta$

$\left[\dfrac{(\sin^4\gamma+\cos^4\gamma)-(\cos^4\gamma+\sin^4\gamma-2\sin^2\gamma\cos^2\gamma)}{\sin^2\gamma\cos^2\gamma}\right]$

$= \alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2$, which is independent of γ and dependent on α, β.

13. (a, b, c, d) : Given, $P(A)=\dfrac{1}{2}\quad\therefore\quad P(\bar{A})=\dfrac{1}{2}$

$P(B)=\dfrac{1}{3}\quad\therefore\quad P(\bar{B})=\dfrac{2}{3}\;;\quad P(C)=\dfrac{1}{4}\quad\therefore\quad P(\bar{C})=\dfrac{3}{4}$

Now, P(none of A, B, C solve the problem) $= P\left(\bar{A}\,\bar{B}\,\bar{C}\right)$

$=\dfrac{1}{2}\cdot\dfrac{2}{3}\cdot\dfrac{3}{4}=\dfrac{1}{4}$

$\therefore$ P(Problem will be solved) $= 1 - P$ (none of them can solve the problem) $=1-\dfrac{1}{4}=\dfrac{3}{4}$

P(only two of them solve the problem)

$=P(AB\bar{C})+P(A\bar{B}C)+P(\bar{A}BC)$

$=\dfrac{1}{2}\cdot\dfrac{1}{3}\cdot\dfrac{3}{4}+\dfrac{1}{2}\cdot\dfrac{2}{3}\cdot\dfrac{1}{4}+\dfrac{1}{2}\cdot\dfrac{1}{3}\cdot\dfrac{1}{4}=\dfrac{6}{24}=\dfrac{1}{4}$

P(exactly one of them solve the problem)

$=P(A\overline{BC})+P(\overline{AC}B)+P(\overline{AB}C)$

$=\dfrac{1}{2}\cdot\dfrac{2}{3}\cdot\dfrac{3}{4}+\dfrac{1}{2}\cdot\dfrac{3}{4}\cdot\dfrac{1}{3}+\dfrac{1}{2}\cdot\dfrac{2}{3}\cdot\dfrac{1}{4}=\dfrac{11}{24}$

14. (a, b, c, d) : $\because \Delta = a^2 - (b - c)^2$

$\Rightarrow\quad \Delta = (a + b - c)(a - b + c) = 4(s - c)(s - b)$

$\Rightarrow\quad \sqrt{s(s-a)(s-b)(s-c)}=4(s-c)(s-b)$

$\Rightarrow\quad \sqrt{\dfrac{s(s-a)}{(s-b)(s-c)}}=4\quad\therefore\quad \tan\dfrac{A}{2}=\dfrac{1}{4}$

$\Rightarrow\quad \sin\dfrac{A}{2}=\dfrac{1}{\sqrt{17}}$

Again, $\tan A=\dfrac{2\tan A/2}{1-\tan^2(A/2)}=\dfrac{2\cdot\dfrac{1}{4}}{1-\dfrac{1}{16}}=\dfrac{1}{2}\cdot\dfrac{16}{15}=\dfrac{8}{15}$

$\Rightarrow \cot A=\dfrac{15}{8}$ and $\operatorname{cosec} A=\dfrac{17}{8}$

15. (a, c, d) : Since, A and B are invertible matrices, then A^{-1} and B^{-1} both exist. Also, for every positive integer, A^n and B^n are invertible.

Now, $(AB)^k = A^k B^k$ hold for $k = 2008, 2009, 2010$.

Now, $(AB)^{2008} = A^{2008} B^{2008}$ ...(i)

$(AB)^{2009} = A^{2009} B^{2009}$ ...(ii)

$(AB)^{2010} = A^{2010} B^{2010}$ ...(iii)

From (ii), we get

$A^{2009} B^{2009} = (AB)^{2009} = (AB)^{2008} (AB)$

$\Rightarrow\quad A^{2008} AB^{2008} B = A^{2008} B^{2008} AB$ [Using (i)]

Now, A^{2008} and B are invertible matrices, then from the above equation, we get

$AB^{2008} = B^{2008} A$ ...(iv)

Similarly, from (ii) and (iii), we get

$AB^{2009} = B^{2009} A$ ...(v)

Now, $(AB)\,B^{2008} = AB^{2009} = B^{2009} A$ [Using (v)]

$= B(B^{2008} A) = B\,(AB^{2008})$ [Using (iv)]

$= (BA)\,B^{2008}$

Thus, $(AB)\,B^{2008} = (BA)\,B^{2008}$

Since, B^{2008} is an invertible matrix, then

$AB = BA$

$\Rightarrow\quad (AB)^{-1} = (BA)^{-1}$

$\Rightarrow\quad B^{-1}A^{-1} = A^{-1}B^{-1}$

$\Rightarrow\quad AB^{-1}A^{-1} = (AA^{-1})\,B^{-1}$

$\Rightarrow\quad AB^{-1}A^{-1} = IB^{-1}$

$\Rightarrow\quad AB^{-1}A^{-1} = B^{-1}$

Now, $A^{-2}BA^2 = A^{-2}BAA = A^{-2}ABA$

$= A^{-1}A^{-1}ABA = A^{-1}BA$

16. (a, b, c, d) : $\therefore$ $P(A_1) = P$ (Selecting 4 and two from the numbers 5 to 11) $=\dfrac{{}^1C_1\times{}^7C_2}{{}^{11}C_3}=\dfrac{7}{55}$

$P(A_2) = P$(Selecting number 7 and two from the numbers 1 to 6) $=\dfrac{1\times{}^6C_2}{{}^{11}C_3}=\dfrac{1}{11}$

and $P(A_1\cap A_2) = P$(Selecting 4 and 7 and one between the number 4 and 7)

$=\dfrac{{}^1C_1\times{}^1C_1\times{}^2C_1}{{}^{11}C_3}=\dfrac{2\times 6}{11\times 10\times 9}=\dfrac{2}{165}$

$\therefore\quad P\left(\dfrac{A_1}{A_2}\right)=\dfrac{P(A_1\cap A_2)}{P(A_2)}=\dfrac{2}{165}\times\dfrac{11}{1}=\dfrac{2}{15}$

and $P\left(\dfrac{A_2}{A_1}\right)=\dfrac{P(A_1\cap A_2)}{P(A_1)}=\dfrac{2}{165}\times\dfrac{55}{7}=\dfrac{2}{21}$

17. (3) :

$\because\quad \dfrac{-\pi}{2}\le\sin^{-1}x\le\dfrac{\pi}{2}\ \forall -1\le x\le 1$

$\therefore\quad 0\le(\sin^{-1}x)^2+(\sin^{-1}y)^2+(\sin^{-1}z)^2\le\dfrac{3\pi^2}{4}$

$\therefore\quad (\sin^{-1}x)^2+(\sin^{-1}y)^2+(\sin^{-1}z)^2=\dfrac{3\pi^2}{4}$ is possible if $x, y, z\in\{-1, 1\}$

$\therefore$ Possible values of $x - y + z$ from the ordered triplet (x, y, z) are as follows

(x, y, z)	$x - y + z$	(x, y, z)	$x - y + z$
$(-1, -1, -1)$	-1	$(1, 1, 1)$	1
$(-1, 1, 1)$	-1	$(1, -1, -1)$	1
$(1, -1, 1)$	3	$(-1, 1, -1)$	-3
$(1, 1, -1)$	-1	$(-1, -1, 1$	1

$\therefore$ Set of values of $x - y + z$ is $\{\pm 1, \pm 3\}$

18. (6) : $n(S)$ = The number of ways in which papers of 5 students can be checked by the 5 teachers = 5^5

$n(A)$ = choosing two teachers out of 5 × the number of ways in which 5 papers can be checked by exactly 2 teachers

$= {}^5C_2 \times (2^5 - 2) = 300$

$\therefore$ Required probability $= \dfrac{n(A)}{n(S)} = \dfrac{300}{5^5} = \dfrac{12}{125} = m$ (given)

Hence, $\dfrac{125m}{2} = 6$

19. (5) : $A^3 = A \cdot A^2 = A(5A - 7I)$ $(\because A^2 = 5A - 7I)$

$= 5A^2 - 7A$

$= 5(5A - 7I) - 7A = 25A - 35I - 7A = 18A - 35I$

Now, $A^4 = A \cdot A^3 = A(18A - 35I)$

$= 18A^2 - 35A = 18(5A - 7I) - 35A = 55A - 126I$

Finally, $A^5 = A \cdot A^4 = A(55A - 126I)$

$= 55A^2 - 126A = 55(5A - 7I) - 126A$

$= 149A - 385I = aA + bI$

$\therefore$ $a = 149$ and $b = -385$

$\Rightarrow$ $a + \dfrac{3b}{7} + 21 = 149 + \dfrac{3 \times (-385)}{7} + 21 = 5$

PAPER-2

1. (a, b, c) : $\theta = \tan^{-1}(2\tan^2\theta) - \tan^{-1}((1/3)\tan\theta)$

$\Rightarrow$ $\tan\theta = \dfrac{2\tan^2\theta - (1/3)\tan\theta}{1 + (2/3)\tan^3\theta}$

$\Rightarrow$ $\tan\theta\left[\dfrac{2\tan\theta - (1/3)}{1 + (2/3)\tan^3\theta} - 1\right] = 0$

which is true if $\tan\theta = 0$ or $\dfrac{2\tan\theta - (1/3)}{1 + (2/3)\tan^3\theta} = 1$

$\Rightarrow$ $\tan^3\theta - 3\tan\theta + 2 = 0$

$\Rightarrow$ $(\tan\theta - 1)^2(\tan\theta + 2) = 0$

which holds if $\tan\theta = 1$ or $\tan\theta = -2$

2. (a, c, d) : Given, $A^T = A$ and $B^T = -B$

$(ABA^T)^T = AB^TA^T = -(ABA^T)$

So, ABA^T is skew-symmetric matrix.

$AB^T + BA^T = -AB + BA$

$\Rightarrow$ $(BA - AB)^T = (BA)^T - (AB)^T$

$= A^TB^T - B^TA^T$

$= -AB + BA$

So, $AB^T + BA^T$ is symmetric matrix.

$((A + B)(A - B))^T$

$= (A - B)^T(A + B)^T$

$= (A^T - B^T)(A^T + B^T)$

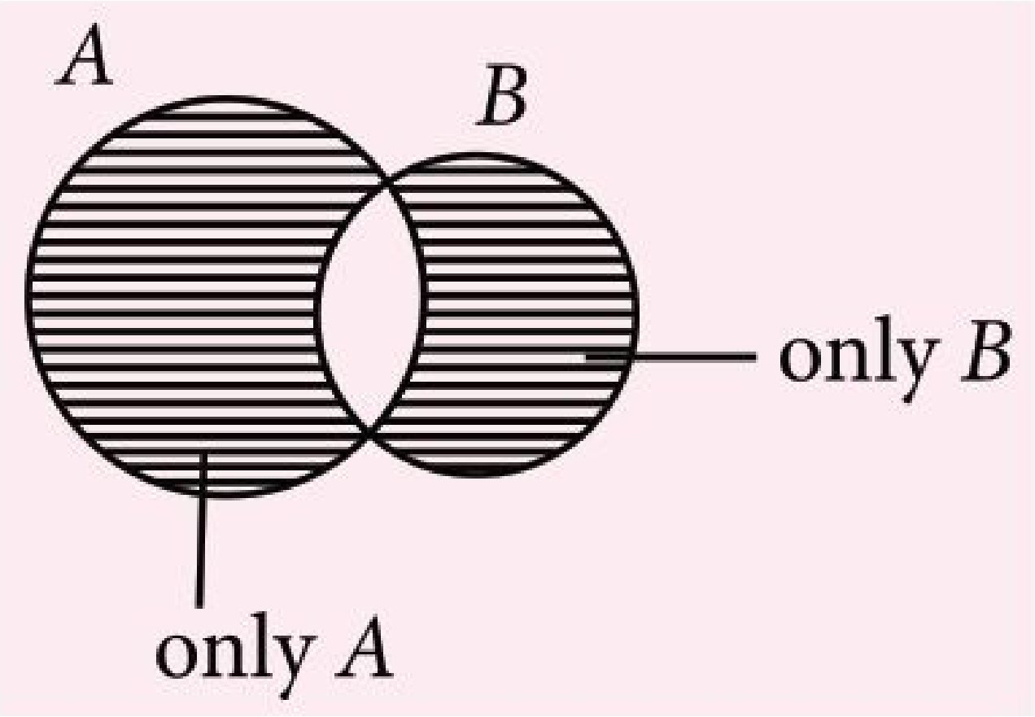
$= (A + B)(A - B)$

So, $(A + B)(A - B)$ is symmetric.

$((A + I)(B - I))^T = (B^T - I)(A^T + I)$

$= -(B + I)(A + I)$

So, $(A + I)(B - I)$ is neither symmetric nor skew symmetric.



3. (a, b, c, d) : Probability that exactly one of them occurs

$= P(\text{only } A \text{ or only } B)$

$= P(\text{only } A) + P(\text{only } B)$

$= P(A \cap B') + P(A' \cap B)$ (option (d))

$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$ (option (a))

$= P(A) + P(B) - P(A \cap B) - P(A \cap B)$

$= P(A \cup B) - P(A \cap B)$ (option (b))

$= 1 - P(A \cup B)' - P(A \cap B)$

$= 1 - P(A' \cap B') - P(A \cap B)$

$= 1 - P(A \cap B) - P(A' \cap B')$

$= P(A' \cup B') - P(A' \cap B')$

$= P(A') + P(B') - P(A' \cap B') - P(A' \cap B')$

$= P(A') + P(B') - 2P(A' \cap B')$ (option (c))

4. (a, b, c, d) : $\because$ $\cos^7 x \le \cos^2 x$ ...(i)

and $\sin^4 x \le \sin^2 x$ ...(ii)

Adding (i) and (ii), we get

$\cos^7 x + \sin^4 x \le \cos^2 x + \sin^2 x = 1$

Equality holds only, if

$\cos^7 x = \cos^2 x$ and $\sin^4 x = \sin^2 x$

$\Rightarrow$ $\cos^2 x(1 - \cos^5 x) = 0$ and $\sin^2 x(1 - \sin^2 x) = 0$

$\Rightarrow$ $\cos^2 x = 0$ or $\cos^5 x = 1$ and $\sin^2 x = 0$ or $\sin^2 x = 1$

$\Rightarrow$ $\cos x = 0$ or $\cos x = 1$ and $\sin x = 0$ or $\sin x = \pm 1$

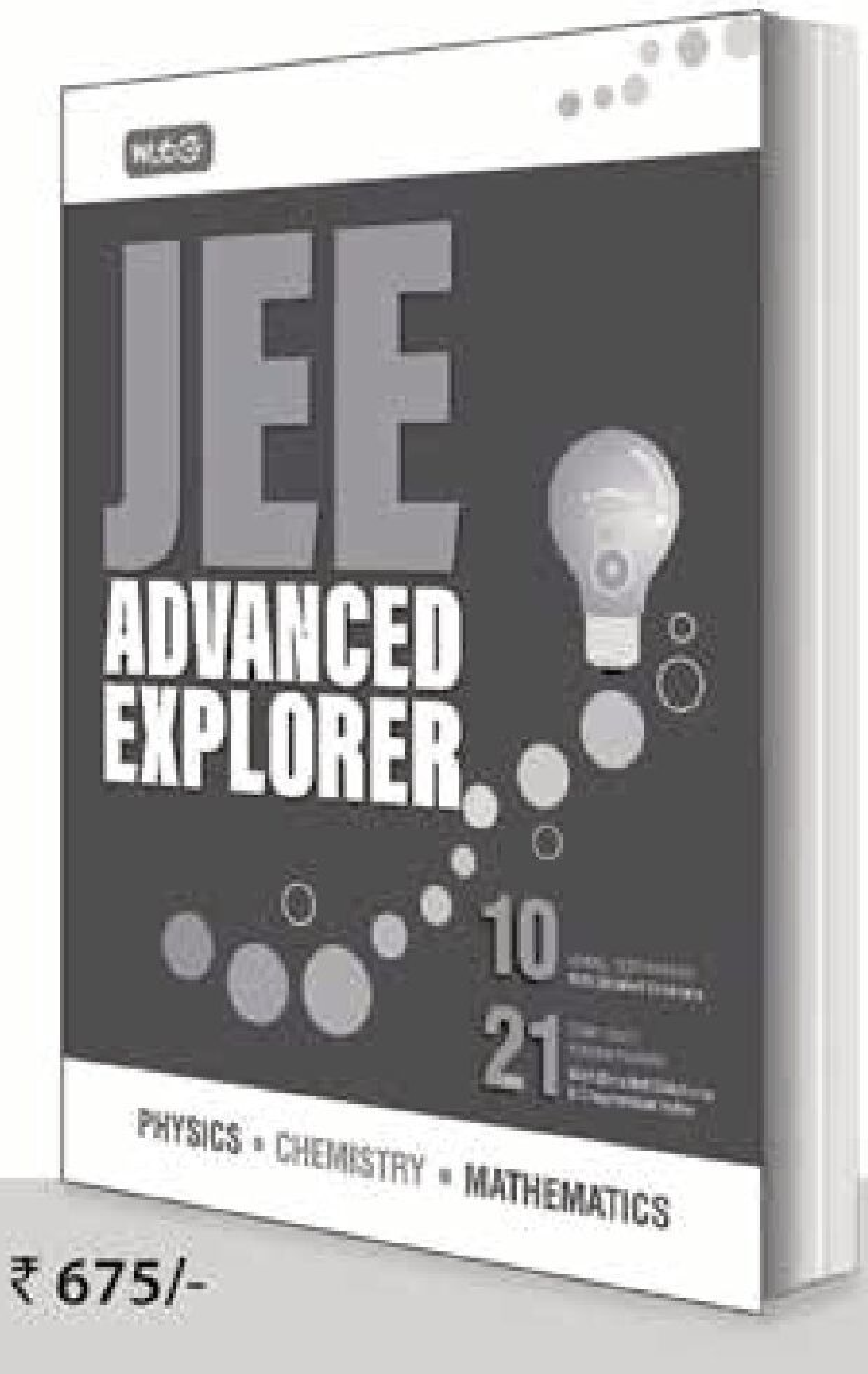
As $x \in (-\pi, \pi)$, $\therefore$ $x = -\dfrac{\pi}{2}, 0, \dfrac{\pi}{2}$ in both the cases.

$\therefore$ Total number of solutions is 3.

5. (a, c): $A_1 = \begin{bmatrix} C_0^2 & 0 \\ 0 & C_1^2 \end{bmatrix}$; $A_2 = \begin{bmatrix} C_1^2 & 0 \\ 0 & C_2^2 \end{bmatrix}$

$A_n = \begin{bmatrix} C_{n-1}^2 & 0 \\ 0 & C_n^2 \end{bmatrix}$

Now, $A_1 + A_2 + ... + A_n$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$$=\begin{bmatrix} C_0^2+C_1^2+.....+C_{n-1}^2 & 0 \\ 0 & C_1^2+C_2^2+.....+C_n^2 \end{bmatrix}$$

$$=\begin{bmatrix} C_0^2+C_1^2+....+C_n^2-C_n^2 & 0 \\ 0 & C_0^2+C_1^2+....+C_n^2-C_0^2 \end{bmatrix}$$

$$=\begin{bmatrix} {}^{2n}C_n-1 & 0 \\ 0 & {}^{2n}C_n-1 \end{bmatrix}$$

So, $k_1 = k_2 = {}^{2n}C_n - 1$

6. (a, b, c) : $P(B)=P(A)P\left(\frac{B}{A}\right)+P(A')P\left(\frac{B}{A}\right)$

$$=P(A)P\left(\frac{B}{A}\right)+P(A')\left[1-P\left(\frac{B'}{A'}\right)\right]$$

$$= \alpha(1 - \alpha) + (1 - \alpha)\,[1 - (1 - \alpha)]$$

$$= \alpha(1 - \alpha) + (1 - \alpha)\,(\alpha) = 2\alpha(1 - \alpha)$$

Again, $P\left(\frac{A'}{B}\right)=\frac{P(B)-P(A\cap B)}{P(B)}$

$$\therefore\ P\left(\frac{A'}{B}\right)=\frac{2\alpha(1-\alpha)-\alpha(1-\alpha)}{2\alpha(1-\alpha)}=\frac{1}{2}$$

Again $P(B') = 1 - P(B) = 1 - (2\alpha - 2\alpha^2)$
$= \alpha^2 + (\alpha^2 - 2\alpha + 1) = \alpha^2 + (\alpha - 1)^2$

Also, $P\left(\frac{A'}{B'}\right)=\frac{P(A'\cap B')}{P(B')}=\frac{1-P(A\cup B)}{1-P(B)}$

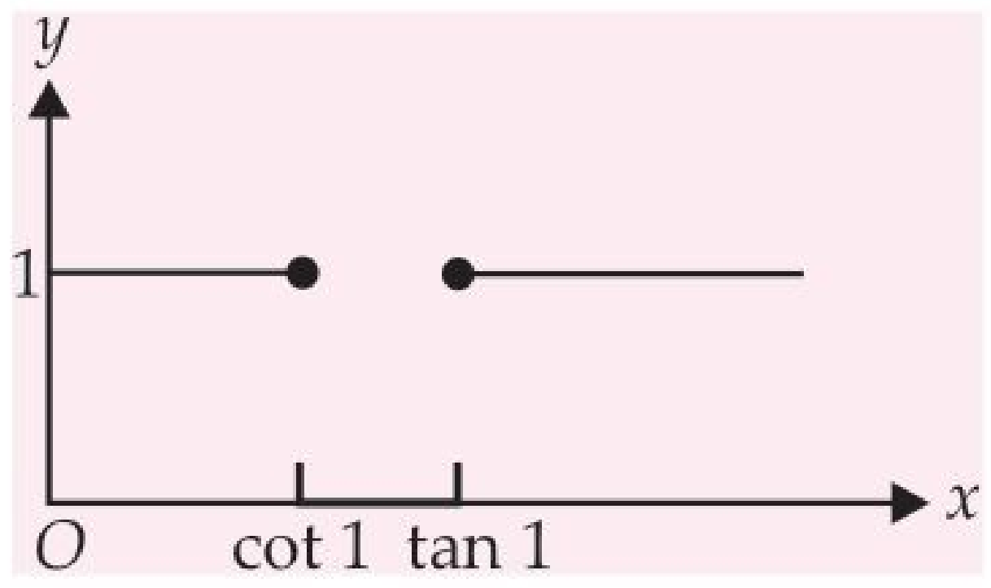
$$=\frac{1-(2\alpha-\alpha^2)}{1-(2\alpha-2\alpha^2)}$$

$$\Rightarrow\quad P\left(\frac{A'}{B'}\right)=\frac{(\alpha-1)^2}{2\alpha^2-2\alpha+1}\neq\left(\frac{\alpha}{1-\alpha}\right)^2$$

7. (5) : The least value is –1.

$$\because\ \frac{-\lambda}{5}=-1\Rightarrow\lambda=5$$

8. (1) : The greatest value of $f(x)$ is 1.



9. (0.35): Consider $\begin{bmatrix}2&1\\1&0\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}1&1\\1&-1\end{bmatrix}=I+B$ (say)

$$\Rightarrow\ A=\begin{bmatrix}2&1\\1&0\end{bmatrix}^n=(I+B)^n\ \Rightarrow\ A=\begin{bmatrix}2&1\\1&0\end{bmatrix}^n$$

$= {}^nC_0I + {}^nC_1B + {}^nC_2B^2 + {}^nC_3B^3 + {}^nC_4B^4 +$ .... (i)

Now $B^2=\begin{bmatrix}1&1\\1&-1\end{bmatrix}\begin{bmatrix}1&1\\1&-1\end{bmatrix}=\begin{bmatrix}2&0\\0&2\end{bmatrix}=2I$

$$\Rightarrow\ \begin{bmatrix}2&1\\1&0\end{bmatrix}^n=({}^nC_0+{}^nC_2\cdot 2+{}^nC_4\cdot 2^2+....)I$$
$$+ ({}^nC_1 + {}^nC_3\cdot 2 + {}^nC_5\,.\,2^2 + ...)B = XI + YB$$

$$\Rightarrow\ \begin{bmatrix}2&1\\1&0\end{bmatrix}^n=\begin{bmatrix}X&0\\0&X\end{bmatrix}+\begin{bmatrix}Y&Y\\Y&-Y\end{bmatrix}=\begin{bmatrix}X+Y&Y\\Y&X-Y\end{bmatrix}$$

Hence a_{12} in $A = Y$
$= {}^nC_1 + {}^nC_3\cdot 2 + {}^nC_5\cdot 2^2 + {}^nC_7.\,2^3 + ...$

$$=\frac{1}{\sqrt{2}}[{}^nC_1.\sqrt{2}+{}^nC_3.(\sqrt{2})^3+{}^nC_5.(\sqrt{2})^5+....]$$

$$=\frac{1}{\sqrt{2}}\left[\frac{(1+\sqrt{2})^n-(1-\sqrt{2})^n}{2}\right]$$

$$\Rightarrow\ \lim_{n\to\infty}\frac{a_{12}}{(\sqrt{2}+1)^n}=\frac{1}{2\sqrt{2}}\lim_{n\to\infty}\left[1-\frac{(1-\sqrt{2})^n}{(1+\sqrt{2})^n}\right]=\frac{1}{2\sqrt{2}}=0.35$$

10. (0.15) : In the matrix A, $a_{22} = X - Y$
$= ({}^nC_0 + {}^nC_2\cdot 2 + {}^nC_4\cdot 2^2 + {}^nC_6\cdot 2^3 + ...)$
$- ({}^nC_1 + {}^nC_3\cdot 2 + {}^nC_5\cdot 2^2 + {}^nC_7\cdot 2^3 + ...)$

$$=\frac{(1+\sqrt{2})^n+(1-\sqrt{2})^n}{2}-\frac{(1+\sqrt{2})^n-(1-\sqrt{2})^n}{2\sqrt{2}}$$

$$=\frac{\sqrt{2}[(1+\sqrt{2})^n+(1-\sqrt{2})^n]-[(1+\sqrt{2})^n-(1-\sqrt{2})^n]}{2\sqrt{2}}$$

$$\Rightarrow\ \lim_{n\to\infty}\frac{a_{22}}{(\sqrt{2}+1)^n}$$

$$=\lim_{n\to\infty}\frac{\sqrt{2}[(1+\sqrt{2})^n+(1-\sqrt{2})^n]-[(1+\sqrt{2})^n-(1-\sqrt{2})^n]}{2\sqrt{2}(1+\sqrt{2})^n}$$

$$=\lim_{n\to\infty}\frac{\sqrt{2}\left[1+\frac{(1-\sqrt{2})^n}{(1+\sqrt{2})^n}\right]-\left[1-\frac{(1-\sqrt{2})^n}{(1+\sqrt{2})^n}\right]}{2\sqrt{2}}=\frac{\sqrt{2}-1}{2\sqrt{2}}=0.15$$

11. (0.28) : $P\left(\frac{E_1}{E}\right)=\frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E)}=\frac{1}{3}\cdot\frac{1}{5}\times\frac{495}{118}=\frac{33}{118}$

$= 0.28$

12. (0.72) : We have, $P\left(\frac{E_2}{E}\right)=\frac{P(E_2\cap E)}{P(E)}=\frac{55}{118}=0.47$

and $P\left(\frac{E_3}{E}\right)=\frac{P(E_3\cap E)}{P(E)}=\frac{P(E_3)\cdot P\left(\frac{E}{E_3}\right)}{P(E)}$

$$=\frac{1}{3}\times\frac{2}{11}\times\frac{495}{118}=\frac{30}{118}=0.25$$

$$\therefore\quad P\left(\frac{E_2}{E}\right)+P\left(\frac{E_3}{E}\right)=0.47+0.25=0.72$$

(13-14) : We have, $S_n = x + x^4 + x^7 + x^{10} + ...$ (upto $2n$ terms)
$C_n = x^2 + x^5 + x^8 + x^{11} + ...$ (upto $2n$ terms)
$T_n = x^3 + x^6 + x^9 + x^{12} + ...$ (upto $2n$ terms)

13. (a) : Clearly $S_n > C_n > T_n$ as 'x' is a proper fraction
$\therefore\quad x > x^2 > x^3$ and so on.

14. (b) : $\lim\limits_{n\to\infty}(S_n+C_n+T_n)=\dfrac{x}{1-x}$

15. (a) : $(A + B)C = (A + B)(A + B)^{-1}(A - B)$
$\Rightarrow\quad (A + B)C = A - B$ ...(i)
$C^T = (A - B)^T((A + B)^{-1})^T$
$= (A + B)((A + B)^T)^{-1}$
$= (A + B)(A - B)^{-1}$ ...(ii)
From (i) and (ii),
$C^T(A + B)\ C = (A + B)(A - B)^{-1}(A - B)$
$= (A + B)$ ...(iii)

16. (b) : Taking transpose in (iii), we get
$C^T(A + B)^T(C^T)^T = (A + B)^T$
$\Rightarrow\quad C^T(A - B)C = A - B$ ...(iv)

17. (5) : Given, $P(A)=\dfrac{2}{5},\ P(A\cap B)=\dfrac{3}{20}$

Now, $P(A' \cup B') = P((A \cap B)') = 1 - P(A \cap B)$
$$=1-\frac{3}{20}=\frac{17}{20}$$
Again, $A \cap (A' \cup B') = A \cap (A \cap B)'$
$= A - ((A \cap B)')' = A - (A \cap B)$

$$\therefore\quad P(A-(A\cap B))=\frac{2}{5}-\frac{3}{20}=\frac{1}{4}=P(\text{only }A)$$

$$\therefore\ \frac{l}{m}=P(A\,|\,(A'\cup B')=\frac{P(A-(A\cap B)}{P(A'\cup B')}\ =\frac{\frac{1}{4}}{\frac{17}{20}}=\frac{5}{17}$$

$\therefore\quad l = 5$

18. (3) : Since $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\dfrac{3\pi}{2}$
and $\sin^{-1}x\le\dfrac{\pi}{2}$ for $x \in [-1, 1]$
Hence above result is possible only when
$\sin^{-1}x=\dfrac{\pi}{2},\sin^{-1}y=\dfrac{\pi}{2},\sin^{-1}z=\dfrac{\pi}{2}$
$\Rightarrow\quad x = 1, y = 1, z = 1$

$$\therefore\quad \sum\frac{(x^{101}+y^{101})(x^{202}+y^{202})}{(x^{303}+y^{303})(x^{404}+y^{404})}$$

$$=\frac{(x^{101}+y^{101})(x^{202}+y^{202})}{(x^{303}+y^{303})(x^{404}+y^{404})}+\frac{(y^{101}+z^{101})(y^{202}+z^{202})}{(y^{303}+z^{303})(y^{404}+z^{404})}+\frac{(z^{101}+x^{101})(z^{202}+x^{202})}{(z^{303}+x^{303})(z^{404}+x^{404})}$$

$$=\frac{(1+1)(1+1)}{(1+1)(1+1)}+\frac{(1+1)(1+1)}{(1+1)(1+1)}+\frac{(1+1)(1+1)}{(1+1)(1+1)}=1+1+1=3$$

19. (4) : Let $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, then $ad - bc = t$ and

$\text{adj}\,A=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$ and

$$|A+t\,\text{adj}\,(A)|=0\ \Rightarrow\ \begin{vmatrix}a+td & b-bt\\ c-ct & d+at\end{vmatrix}=0$$

On simplification, we get
$(a + d)^2 + (t - 1)^2 = 0$
i.e., $a + d = 0$ and $t = 1$
and $|A - t(\text{adj}\cdot A)| = t(1 + t)^2 = 4$

◆

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique solution.

1				8			3	9				1				6			7	4
9	7	2		3			4					5	4	3		8			9	
		5				2	7							6				5	2	
			4		8										5		8			
4	2						8	5				7	8						3	5
			6		3										9		1			
	1	7				9				3			5	8				1		
	4			9		8	5	3		2			1			5		3	4	2
5	9			6				4				6	3			1				8
									6		2									
						3	8						4	6						
									3		4									
9				4			3	8				5				4			3	8
4	7	8		5			9			5		3	7	4		2			6	
		3				5	4			6				9				5	7	
			6		4										6		8			
7	6						8	4				1	3						4	2
			3		7										4		2			
	4	1				7							9	2				7		
	8			7		4	5	2					5			7		1	9	6
2	9			6				3				7	1			8				5

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' names with their valuable feedback will be published in next issue.

JEE WORK OUTS

One or More Than One Option(s) Correct Type

1. If $f_n(\theta) = \tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta) \ldots (1 + \sec 2^n\theta)$, then
(a) $f_2\left(\frac{\pi}{16}\right)=1$ (b) $f_3\left(\frac{\pi}{32}\right)=1$
(c) $f_4\left(\frac{\pi}{64}\right)=1$ (d) $f_5\left(\frac{\pi}{128}\right)=1$

2. Let $f(x)=\max\{2-x,2,1+x\}$. Then $\int_{-1}^{1} f(x)dx =$
(a) 0 (b) 2 (c) 9/2 (d) 3

3. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane containing the lines L_1 and L_2 is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{4\sqrt{2}}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$

4. The value of $\int_{\log\sqrt{\frac{\pi}{2}}}^{\log\sqrt{\pi}} e^{2x}\sec^2\left(\frac{1}{3}e^{2x}\right)dx =$
(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{3\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{3}}$

5. $ABCD$ is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD = 2AC$. If the points D and M represent the complex numbers $1 + i$ and $2 - i$ respectively, then the complex number represented by the point A is
(a) $3-\frac{i}{2}$ (b) $1+\frac{i}{2}$ (c) $1-\frac{3i}{2}$ (d) $3+\frac{i}{2}$

6. The kindergarten teacher has 25 kids in her class. She takes 5 of them at a time to zoological garden as often as she can without taking the same 5 kids more than once. Then the number of visits, the teacher makes to the garden exceeds that of a kid by
(a) ${}^{25}C_5 - {}^{24}C_4$ (b) ${}^{24}C_5$
(c) ${}^{25}C_5 - {}^{24}C_5$ (d) ${}^{24}C_4$

7. If $f(\alpha)=\lim_{x\to 2}(\sin^x\alpha+\cos^x\alpha)^{\frac{1}{(x-2)}}$ for $\alpha\in\left[0,\frac{\pi}{2}\right]$, then
(a) $f(0) = 1$ (b) $f\left(\frac{\pi}{2}\right)=1$
(c) $f(\alpha)=(\cos\alpha)^{\cos^2\alpha}\cdot(\sin\alpha)^{\sin^2\alpha}$ if $\alpha\in\left(0,\frac{\pi}{2}\right)$
(d) $f(\alpha)=\frac{(\sin\alpha)^{\sin^2\alpha}}{(\cos\alpha)^{\cos^2\alpha}}$ if $\alpha\in\left(0,\frac{\pi}{2}\right)$

8. The area between the curves $y = xe^x$ and $y = xe^{-x}$ and the line $x = 1$ is
(a) $2e$ (b) e (c) $2/e$ (d) $1/e$

9. The area bounded by the curve, $y = f(x) = x^4 - 2x^3 + x^2 + 3$, the x-axis and the ordinates corresponding to the minimum of function $f(x)$ is
(a) 1 (b) $\frac{91}{30}$
(c) $\frac{30}{9}$ (d) 4

10. Three lines $px + qy + r = 0$, $qx + ry + p = 0$ and $rx + py + q = 0$ are concurrent, if
(a) $p + q + r = 0$
(b) $p^2 + q^2 + r^2 = pq + qr + rp$
(c) $p^3 + q^3 + r^3 = 3pqr$
(d) none of these

11. If the matrix $A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal, then

(a) $\alpha = \pm\frac{1}{\sqrt{2}}$ (b) $\alpha = \pm\frac{1}{\sqrt{3}}$

(c) $\beta = \pm\frac{1}{\sqrt{6}}$ (d) $\alpha = \pm\sqrt{3}\beta$

12. If $f(x) = \min\{1, x^2, x^3\}$, then
(a) $f(x)$ is continuous
(b) $f(x) > 1 \ \forall\ x > 1$
(c) $f(x)$ is not differentiable
(d) $f(x)$ is not differentiable for two values of x

13. Consider the family of curves $C : y = cx^2$.
(a) The differential equation of C is $xdy - 2ydx = 0$
(b) The differential equation of orthogonal trajectories of C is $x\,dx + 2ydy = 0$
(c) The equation of family of orthogonal trajectories of C is $x^2 + 2y^2 = c$
(d) The equation of family of orthogonal trajectory of C is $2x^2 + y^2 = c$

14. The value of $\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}} =$

(a) $1+\sqrt{5}$ (b) $-1+\sqrt{5}$ (c) $-1+\sqrt{2}$ (d) $1+\sqrt{2}$

15. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x}\,dx$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{2}$ (d) 4π

16. If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j}$ and $\vec{r} \times \vec{a} = \vec{b}$, then possible values of $9\vec{r}$ are

(a) $2\hat{i} + \hat{j} + 7\hat{k}$ (b) $-6\hat{i} + 3\hat{j} + 3\hat{k}$

(c) $2\hat{i} - \hat{j} + 11\hat{k}$ (d) $-2\hat{i} + \hat{j} + 7\hat{k}$

17. The plane $ax + by = 0$ is rotated through an angle α about its line of intersection with the plane $z = 0$. Then the equation to the plane in new position is/are

(a) $ax + by + z\sqrt{a^2+b^2}\tan\alpha = 0$

(b) $ax + by - z\sqrt{a^2+b^2}\tan\alpha = 0$

(c) $ax + by + z\sqrt{a^2+b^2}\cot\alpha = 0$

(d) $ax + by - z\sqrt{a^2+b^2}\cot\alpha = 0$

18. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is

(a) $2\sqrt{2}$ (b) 3 (c) -3 (d) $-2\sqrt{2}$

19. The integral

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2}\,dx$$

is equal to

(a) $\frac{-1}{1+\cot^3 x} + C$ (b) $\frac{1}{3(1+\tan^3 x)} + C$

(c) $\frac{-1}{3(1+\tan^3 x)} + C$ (d) $\frac{1}{1+\cot^3 x} + C$

20. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is

(a) $\frac{3}{4}$ (b) $\frac{3}{10}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

Comprehension Type

Paragraph for Q. No. 21 to 23

Consider the function $f(x)$, a fourth degree polynomial such that $\lim_{x\to -1} \frac{f(x)}{(x+1)^3} = 1$ and $f'''(0) = 0$.

21. The maximum value of $f(x)$ is

(a) $\frac{13}{4}$ (b) $\frac{17}{4}$ (c) $\frac{23}{4}$ (d) $\frac{27}{4}$

22. The subnormal of the curve $y = f(x)$ where it cuts the y-axis is

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{1}{2}$

23. The number of inflectional points on the curve $y = f(x)$ is
(a) 0 (b) 1 (c) 2 (d) 3

Paragraph for Q. No. 24 and 25

The vertices of ΔABC lies on a rectangular hyperbola such that the orthocentre of triangle is (3, 2) and the asymptotes of rectangular hyperbola are parallel to coordinate axis. The two perpendicular tangents of the hyperbola intersect at the point (1, 1).

24. Equation of asymptotes of the hyperbola is
(a) $(x-1)(y-1)+\lambda=0$
(b) $(x-1)(y-1)=0$
(c) $xy+1=x-y$ (d) $(x+1)(y+1)=0$

25. The equation of hyperbola is
(a) $(x-1)(y-1)=2$ (b) $y(x-1)=x+1$
(c) $x(y-1)=y+1$ (d) all of these

Paragraph for Q. No. 26 to 28

Let $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and non-zero vector $\vec{c}$ are such that $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$.

26. The vector $\vec{c}$ may be given as
(a) $4\hat{i}+2\hat{j}+4\hat{k}$ (b) $4\hat{i}-2\hat{j}+4\hat{k}$
(c) $\hat{i}+\hat{j}+\hat{k}$ (d) $\hat{i}-4\hat{j}+\hat{k}$

27. Volume (in cubic units) of parallelepiped whose adjacent sides are given by $\vec{a}, \vec{b}, \vec{b}\times\vec{c}$ is
(a) 18 (b) 54 (c) 12 (d) 36

28. A vector along the bisector of angle between the vectors $\vec{b}$ and $\vec{c}$ is
(a) $(2+\sqrt{3})\hat{i}+(1-\sqrt{3})\hat{j}+(2+\sqrt{3})\hat{k}$
(b) $(2-\sqrt{3})\hat{i}+(\sqrt{3}+1)\hat{j}+(2-\sqrt{3})\hat{k}$
(c) $(2+\sqrt{3})\hat{i}+(1-\sqrt{3})\hat{j}-(2+\sqrt{3})\hat{k}$
(d) $(2+\sqrt{3})\hat{i}-(1-\sqrt{3})\hat{j}+(2+\sqrt{3})\hat{k}$

Matrix Match Type

29. Match the following :

	Column-I		Column-II
(A)	Let $f(x)=[x-1]+[1-x]$, $[x]$ is the greatest integer function, a is an integer, then	(P)	continuous at $x=a$
(B)	Let f be as in (A) but a is not an integer, then	(Q)	$\lim_{x\to 0} f(x)$ does not exist
(C)	Let $f(x)=\cot x$, then	(R)	$f(a)=0$
(D)	If $f(x)=\dfrac{a^{\cot x}-a^{\cos x}}{\cot x-\cos x}$, then	(S)	$\lim_{x\to\pi/2} f(x)=\log a$

30. Match the following :

	Column-I		Column-II
(A)	If $\int x^2 d(\tan^{-1}x)dx=x-f(x)+c$ then $f(1)$ is equal to	(P)	$\dfrac{1}{8}$
(B)	If $\int x^2e^{2x}dx=e^{2x}f(x)+c$, then the minimum value of $f(x)$ is	(Q)	0
(C)	If $\int\dfrac{(x^4+1)}{x(x^2+1)^2}dx=a\log\lvert x\rvert+\dfrac{b}{x^2+1}+c$ then $a-b$ is equal to	(R)	$\dfrac{\pi}{4}$

31. Match the following :

	Column-I		Column-II
(A)	Let $\lvert A\rvert=\lvert a_{ij}\rvert_{3\times 3}\neq 0$. Each element a_{ij} is multiplied by k^{i-j}. Let $\lvert B\rvert$ the resulting determinant, where $k_1\lvert A\rvert+k_2\lvert B\rvert=0$. Then $k_1+k_2=$	(P)	0
(B)	The maximum value of a third order determinant each of its entries are ± 1 equals	(Q)	4
(C)	If $\begin{vmatrix}1&\cos\alpha&\cos\beta\\\cos\alpha&1&\cos\gamma\\\cos\beta&\cos\gamma&1\end{vmatrix}=\begin{vmatrix}0&\cos\alpha&\cos\beta\\\cos\alpha&0&\cos\gamma\\\cos\beta&\cos\gamma&0\end{vmatrix}$ then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=$	(R)	1
(D)	If $\begin{vmatrix}x^2+x&x+1&x-2\\2x^2+3x-1&3x&3x-3\\x^2+2x+3&2x-1&2x-1\end{vmatrix}=Ax+B$, then $A+2B$ is	(S)	$\begin{vmatrix}1&2\\2&4\end{vmatrix}$

Numerical Answer Type

32. If $\int\dfrac{(\sqrt{x})^5}{(\sqrt{x})^7+x^6}dx=a\log\left(\dfrac{x^\lambda}{x^\lambda+1}\right)+C$, then $a\lambda=$ ________.

33. Let $f(x)$ be a polynomial satisfying $\lim_{x\to\infty}\dfrac{x^2f(x)}{2x^5+3}=6$, $f(1)=3$, $f(3)=7$ and $f(7)=15$. Then the value of $\left[\dfrac{\lvert f(0)\rvert}{30}\right]$, where $[\cdot]$ represents the greatest integer function, is ______.

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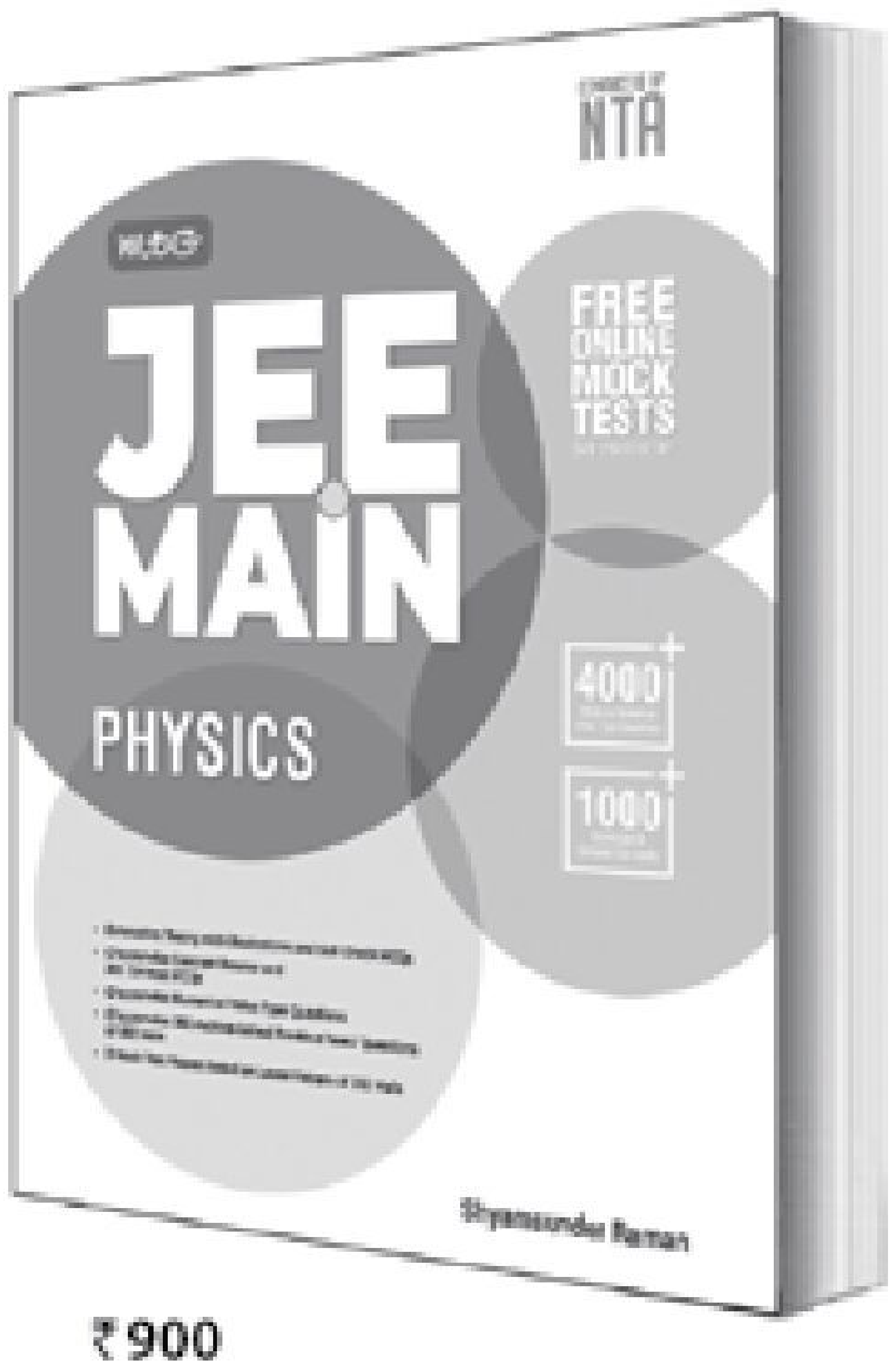
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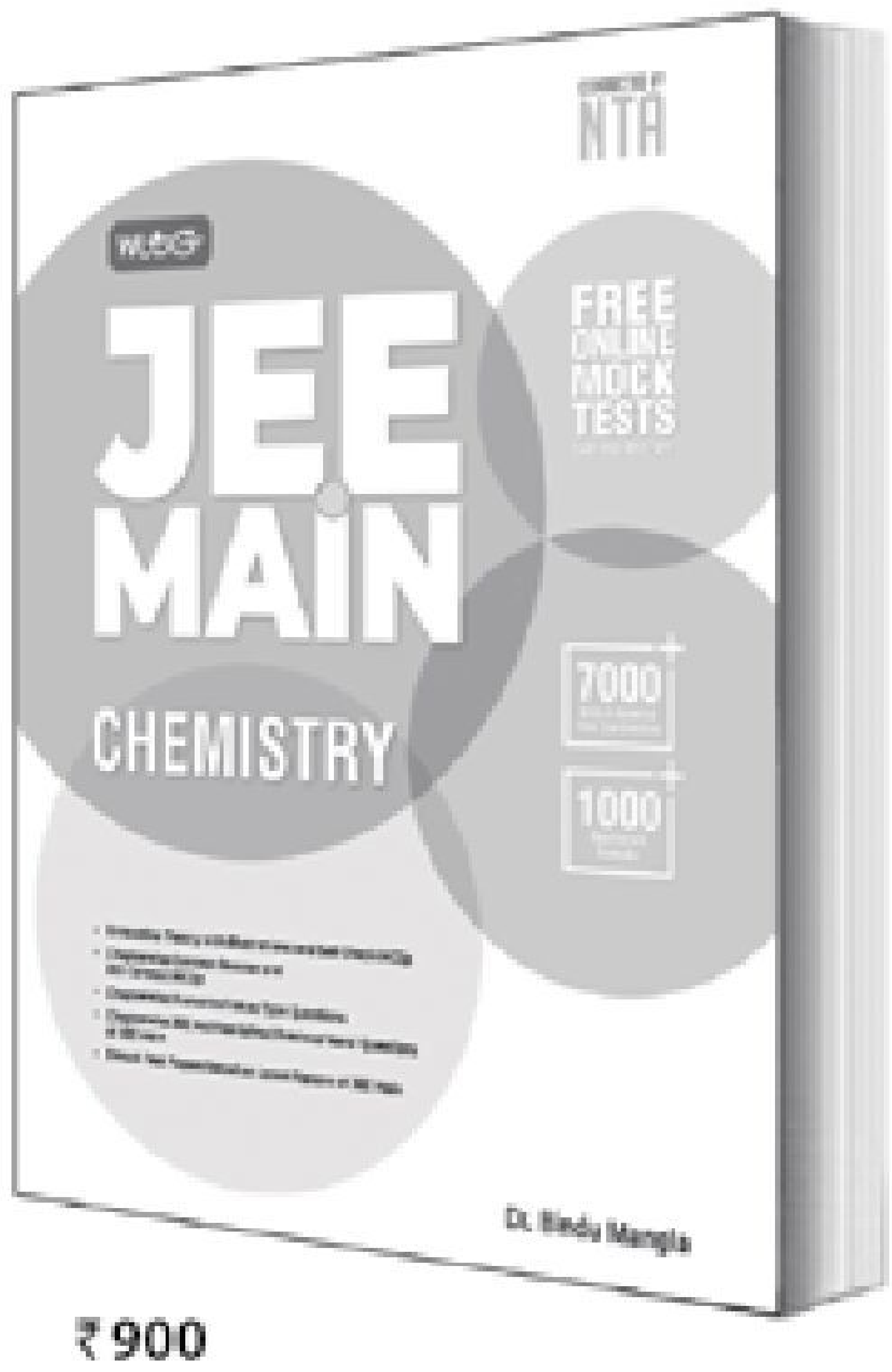
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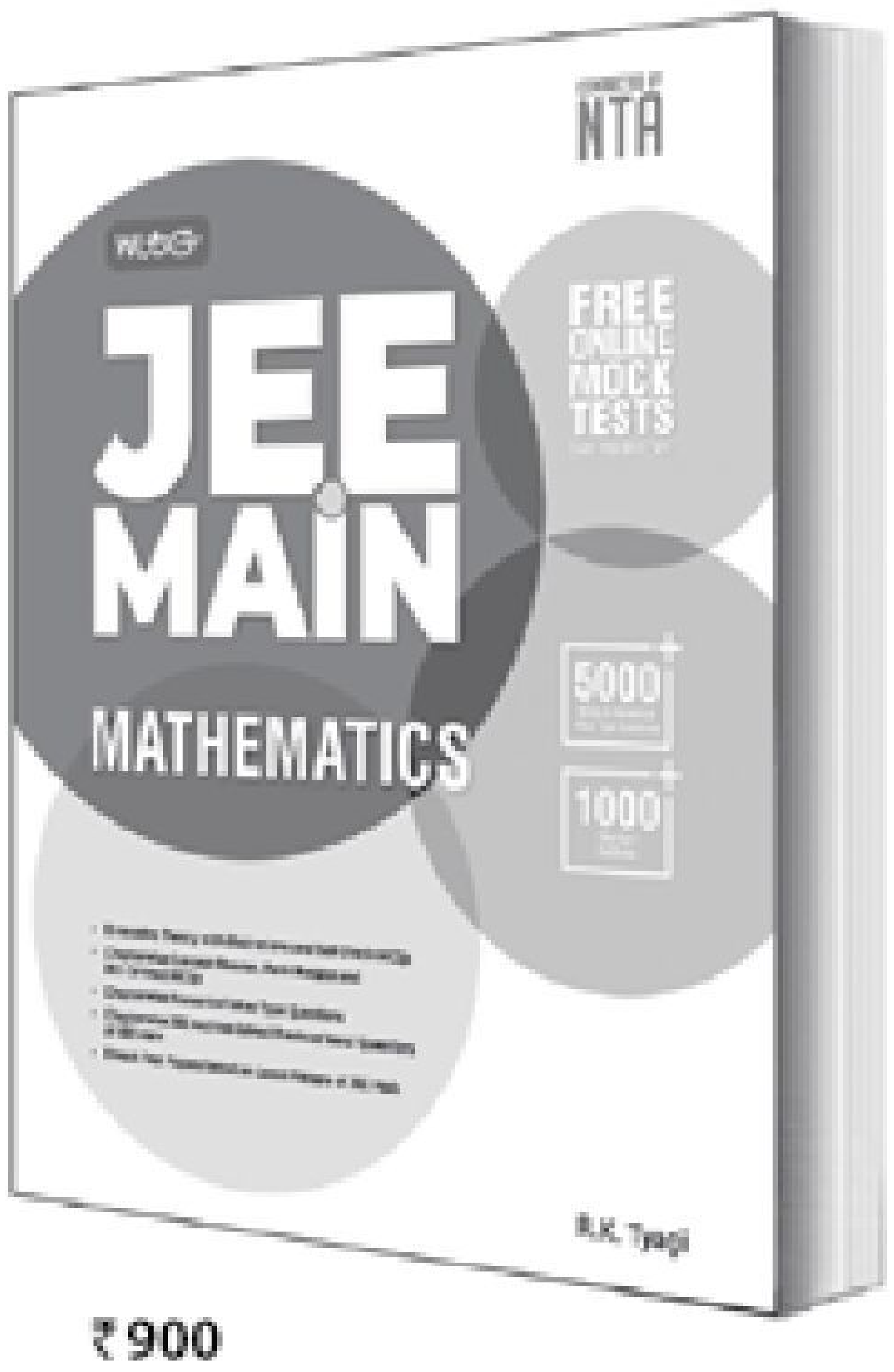
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34. If $\sum_{r=1}^{10}\tan^{-1}\left(\frac{3}{9r^2+3r-1}\right)=\cot^{-1}\left(\frac{m}{n}\right)$ (where m and n are coprime), then the value of $(2m + n)$ is ______.

35. The curve passing through the point (1, 1) satisfies the differential equation $\frac{dy}{dx}+\frac{\sqrt{(x^2-1)(y^2-1)}}{xy}=0.$

If the curve passes through the point $(\sqrt{2},k)$, then the value of $[k]$ is (where $[\cdot]$ represents greatest integer function) ______.

36. If $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and $|A| = 4$, then α is equal to ______.

37. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is ______.

38. Let $\cos(\alpha+\beta)=\frac{4}{5}$ and $\sin(\alpha-\beta)=\frac{5}{13}$, where $0\le\alpha,\ \beta\le\frac{\pi}{4}$. Then $\tan 2\alpha$ is equal to ______.

39. Let $f : R \to R$ be a continuous odd function, which vanishes exactly at one point and $f(1)=\frac{1}{2}$. Suppose that $F(x)=\int_{-1}^{x} f(t)dt$ for all $x \in [-1, 2]$ and $G(x)=\int_{-1}^{x} t\,|f(f(t))|\,dt$ for all $x \in [-1, 2]$. If $\lim_{x\to 1}\frac{F(x)}{G(x)}=\frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is ______.

40. If the mean deviation of numbers 1, $1 + d$, $1 + 2d$, ..., $1 + 100d$ from their mean is 255, then d is equal to ______.

SOLUTIONS

1. (a, b, c, d) : $f_0(\theta)=\tan\frac{\theta}{2}(1+\sec\theta)$

$$=\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\left(1+\frac{1}{\cos\theta}\right)=\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\frac{(\cos\theta+1)}{\cos\theta}$$

$$=\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos\theta}=\frac{\sin\theta}{\cos\theta}=\tan\theta$$

$f_1(\theta) = \tan\theta\,(1 + \sec 2\theta)=\frac{\sin\theta}{\cos\theta}\frac{(\cos 2\theta+1)}{\cos 2\theta}=\tan 2\theta$

Likewise $f_n(\theta) = \tan(2^n\theta)$

$f_2\left(\frac{\pi}{16}\right)=\tan\left(2^2\cdot\frac{\pi}{16}\right)=1, f_3\left(\frac{\pi}{32}\right)=\tan\left(2^3\cdot\frac{\pi}{32}\right)=1,$

$f_4\left(\frac{\pi}{64}\right)=\tan\left(2^4\cdot\frac{\pi}{64}\right)=1$ and

$f_5\left(\frac{\pi}{128}\right)=\tan\left(2^5\cdot\frac{\pi}{128}\right)=1$

2. (c) : In $(-1, 0)$, $2 - x$ is maximum and in $(0, 1)$, 2 is maximum.

$$\therefore\quad \int_{-1}^{1} f(x)dx=\int_{-1}^{0}(2-x)dx+\int_{0}^{1}2dx$$

$$=\left[2x-\frac{x^2}{2}\right]_{-1}^{0}+2=\frac{5}{2}+2=\frac{9}{2}$$

3. (c) : A plane passing through the intersection of the given planes is

$(2x - 2y + 3z - 2) + \lambda(x - y + z + 1) = 0$

$\Rightarrow\quad (\lambda + 2)x - (2 + \lambda)y + (\lambda + 3)z + (\lambda - 2) = 0$

The plane is having infinite number of solutions with $x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$.

$$\therefore\quad \begin{vmatrix}(\lambda+2) & -(\lambda+2) & (\lambda+3)\\ 1 & 2 & -1\\ 3 & -1 & 2\end{vmatrix}=0$$

$\Rightarrow\quad (\lambda + 2)(4 - 1) + (\lambda + 2)(2 + 3) + (\lambda + 3)(-1 - 6) = 0$

$\Rightarrow\quad \lambda = 5$

$\therefore$ The equation of the plane becomes

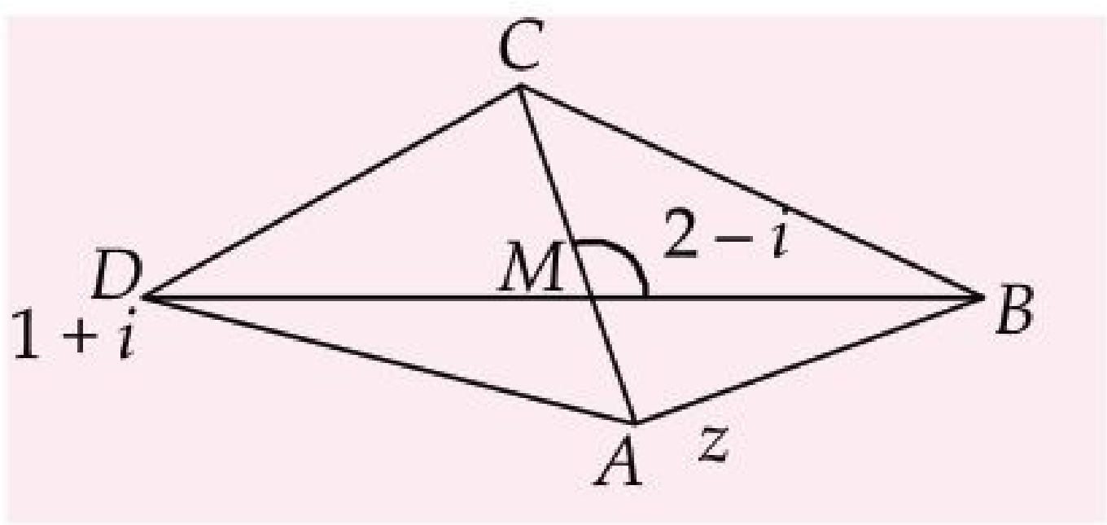
$7x - 7y + 8z + 3 = 0$

The perpendicular distance from origin is

$$\frac{3}{\sqrt{7^2+7^2+8^2}}=\frac{3}{\sqrt{162}}=\frac{3}{9\sqrt{2}}=\frac{1}{3\sqrt{2}}$$

4. (a) : Let $I=\int_{\log\sqrt{\frac{\pi}{2}}}^{\log\sqrt{\pi}} e^{2x}\sec^2\left(\frac{1}{3}e^{2x}\right)dx$

$$=\frac{3}{2}\tan\left(\frac{1}{3}e^{2x}\right)_{\ln\sqrt{\frac{\pi}{2}}}^{\ln\sqrt{\pi}}=\frac{3}{2}\left[\tan\frac{\pi}{3}-\tan\frac{\pi}{6}\right]=\sqrt{3}$$



5. (a, c) : MA is obtained by rotation of MD through $\pm\frac{\pi}{2}$ and dividing by 2.

$$\therefore \quad z-(2-i)=\pm\frac{i(1+i-(2-i))}{2}=\pm\left(1+\frac{i}{2}\right)$$

$$\therefore \quad z=2-i\pm\left(1+\frac{i}{2}\right)=3-\frac{i}{2},1-\frac{3i}{2}$$

6. (a, b) : Number of visits teacher makes
= Number of ways of selecting 5 students from 25
= $^{25}C_5$
Number of visits a particular kid makes
= Number of ways of selecting 4 students accompanying him/her = $^{24}C_4$
So, difference between the number of trips made by teacher and kid = $^{25}C_5 - {}^{24}C_4 = {}^{24}C_5$

7. (a, b, c) : $f(\alpha)=\lim\limits_{x\to 2}(\sin^x\alpha+\cos^x\alpha)^{\frac{1}{(x-2)}}$ (1^∞ form)

$$=\begin{cases} e^{\lim\limits_{x\to 2}\frac{\sin^x\alpha+\cos^x\alpha-1}{x-2}}, & \alpha\in\left(0,\frac{\pi}{2}\right)\\ 1, & \alpha=0,\frac{\pi}{2}\end{cases}$$

$$=e^{\lim\limits_{x\to 2}\frac{\sin^x\alpha+\cos^x\alpha-(\sin^2\alpha+\cos^2\alpha)}{x-2}}$$

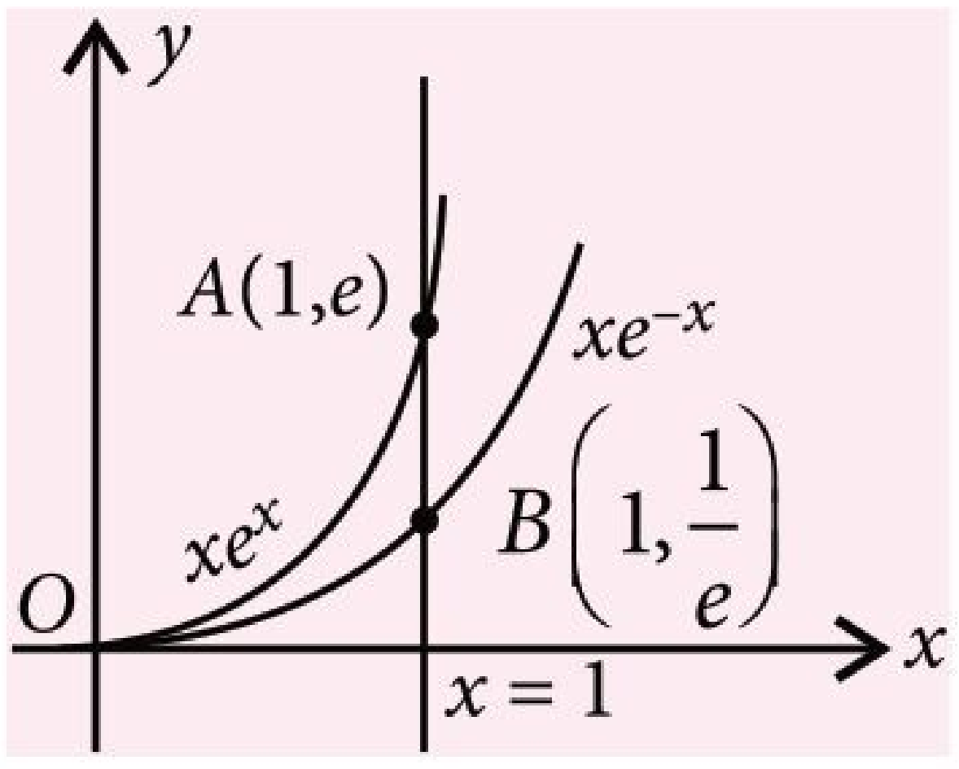
$$=e^{\lim\limits_{x\to 2}\frac{\sin^2\alpha(\sin^{x-2}\alpha-1)+\cos^2\alpha(\cos^{x-2}\alpha-1)}{x-2}}$$

$$=e^{\sin^2\alpha\log_e\sin\alpha+\cos^2\alpha\log_e\cos\alpha}=e^{\log_e(\sin\alpha)^{\sin^2\alpha}+\log_e(\cos\alpha)^{\cos^2\alpha}}$$

$$=e^{\log_e(\sin\alpha)^{\sin^2\alpha}(\cos\alpha)^{\cos^2\alpha}}=(\sin\alpha)^{\sin^2\alpha}(\cos\alpha)^{\cos^2\alpha}$$

$$\therefore\ f(x)=\begin{cases}(\cos\alpha)^{\cos^2\alpha}\cdot(\sin\alpha)^{\sin^2\alpha}, & \alpha\in\left(0,\frac{\pi}{2}\right)\\ 1, & \alpha=0,\frac{\pi}{2}\end{cases}$$

8. (c) : The line $x = 1$ meets the curves in $A(1, e)$ and $B\left(1,\frac{1}{e}\right)$. Both the curves pass through origin.



The required area

$$=\int_0^1(xe^x-xe^{-x})dx$$

$$=\frac{2}{e}\text{ sq.units}$$

9. (b): We have , $f(x)=x^4-2x^3+x^2+3$

$\therefore\quad f'(x)=4x^3-6x^2+2x$

$f'(x)=0 \Rightarrow 4x^3-6x^2+2x=0 \Rightarrow x=0,1,\frac{1}{2}$

$f''(x)=12x^2-12x+2=2(6x^2-6x+1)$

$f''(x)>0 \Rightarrow$ Minimum exists at $x=0$ and $x=1$

$f''(x)<0 \Rightarrow$ Maximum exists at $x=\frac{1}{2}$

$\Rightarrow$ The curve is bounded by the ordinates $x = 0$ and $x = 1$

$\therefore$ Required area $=\int_0^1(x^4-2x^3+x^2+3)dx$

$$=\left(\frac{x^5}{5}-\frac{x^4}{2}+\frac{x^3}{3}+3x\right)_0^1=\frac{91}{30}\text{ sq.units}$$

10. (a, c)

11. (a, c, d): $A\cdot A^T = I$

$\therefore\quad 2\beta^2=\gamma^2,\ \alpha^2=\beta^2+\gamma^2,\ \alpha^2+\beta^2+\gamma^2=1$

$\Rightarrow\quad \frac{\alpha^2}{3}=\frac{\beta^2}{1}=\frac{\gamma^2}{2}=K\Rightarrow\alpha=\pm\sqrt{3}\beta$

So, $\alpha^2+\beta^2+\gamma^2=1$

$\Rightarrow\quad \beta=\pm\frac{1}{\sqrt{6}},\ \alpha=\pm\frac{1}{\sqrt{2}}$

12. (a, c) : $f(x)=\begin{cases}x^3, & x\le 1\\ 1, & x>1\end{cases}$

$\therefore\quad f(x)$ is continuous, but not differentiable at $x = 1$.

13. (a, b, c) : $\frac{dy}{dx}=2cx$

$$\therefore\ \ y=cx^2\Rightarrow y=\left(\frac{1}{2x}\frac{dy}{dx}\right)x^2\Rightarrow y=\frac{x}{2}\frac{dy}{dx} \qquad \text{...(i)}$$

For orthogonal trajectory, replace $\frac{dy}{dx}$ by $\frac{-dx}{dy}$, then (i) becomes

$$y=\frac{x}{2}\left(\frac{-dx}{dy}\right)\Rightarrow 2y\,dy+x\,dx=0$$

On solving we get, $x^2 + 2y^2 = c$

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1. (d) **2.** (a) **3.** (d) **4.** (b) **5.** (d)
6. (a) **7.** (a,b,c) **8.** (a,c) **9.** (c)
10. (a, b, c, d) **11.** (a) **12.** (b,d) **13.** (c)
14. (b) **15.** (d) **16.** (b) **17.** (1) **18.** (3)
19. (1.25) **20.** (1)

14. (b): $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\frac{r}{\sqrt{n^2+r^2}}=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\frac{\frac{r}{n}}{\sqrt{1+\left(\frac{r}{n}\right)^2}}$

$$=\int_0^2\frac{x}{\sqrt{1+x^2}}dx=\frac{1}{2}\cdot\left(2\sqrt{1+x^2}\right)_0^2=\sqrt{5}-1$$

15. (a): Let $I=\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^x}dx$ ...(i)

Changing x to $-\pi/2+\pi/2-x=-x$, we have

$I=\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^{-x}}dx$ ...(ii)

Adding (i) & (ii), we get

$$2I=\int_{-\pi/2}^{\pi/2}\sin^2 x\left\{\frac{1}{1+2^x}+\frac{1}{1+2^{-x}}\right\}dx=\int_{-\pi/2}^{\pi/2}\sin^2 x\,dx$$

$=2\int_0^{\pi/2}\sin^2 x\,dx$ (As the integral function is even)

$\therefore\ I=\frac{\pi}{4}$

16. (b, c, d) : $\vec{a}\cdot\vec{b}=2-2=0$

The general solution of the vector equation

$\vec{r}\times\vec{a}=\vec{b},\ \vec{a}\cdot\vec{b}=0$ is $\vec{r}=\frac{\vec{a}\times\vec{b}}{\vec{a}\cdot\vec{a}}+\alpha\vec{a}$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&-1&2\\1&2&0\end{vmatrix}$

$=-4\hat{i}+2\hat{j}+5\hat{k}$ and $\vec{a}\cdot\vec{a}=9$

$\therefore\quad\vec{r}=\frac{-4\hat{i}+2\hat{j}+5\hat{k}}{9}+\alpha\,(2\hat{i}-\hat{j}+2\hat{k})$

$\Rightarrow\quad 9\vec{r}=-4\hat{i}+2\hat{j}+5\hat{k}+9\alpha(2\hat{i}-\hat{j}+2\hat{k})$

$\alpha=\frac{1}{9}\Rightarrow 9\vec{r}=-2\hat{i}+\hat{j}+7\hat{k}$,

$\alpha=\frac{1}{3}\Rightarrow 9\vec{r}=2\hat{i}-\hat{j}+11\hat{k}$

$\alpha=-\frac{1}{9}\Rightarrow 9\vec{r}=-6\hat{i}+3\hat{j}+3\hat{k}.$

17. (a, b): Given planes are $ax+by=0$ ...(i)

and $z=0$ ...(ii)

$\therefore$ Equation of any plane passing through the line of intersection of planes (i) and (ii) may be taken as

$ax+by+kz=0$ ...(iii)

The direction cosines of a normal to the plane (iii) are

$$\frac{a}{\sqrt{a^2+b^2+k^2}},\frac{b}{\sqrt{a^2+b^2+k^2}},\frac{k}{\sqrt{a^2+b^2+k^2}}$$

The direction cosines of a normal to the plane (i) are

$$\frac{a}{\sqrt{a^2+b^2}},\frac{b}{\sqrt{a^2+b^2}},0$$

Since the angle between the planes (i) and (ii) is α

$$\therefore\ \cos\alpha=\frac{a\cdot a+b\cdot b+k\cdot 0}{\sqrt{a^2+b^2+k^2}\sqrt{a^2+b^2}}=\sqrt{\frac{a^2+b^2}{a^2+b^2+k^2}}$$

$\Rightarrow\quad k^2\cos^2\alpha=a^2(1-\cos^2\alpha)+b^2(1-\cos^2\alpha)$

$\Rightarrow\quad k^2=\frac{(a^2+b^2)\sin^2\alpha}{\cos^2\alpha}\Rightarrow k=\pm\sqrt{a^2+b^2}\ \tan\alpha$

Putting in (iii) we get equation of plane as;

$ax+by\pm z\sqrt{a^2+b^2}\ \tan\alpha=0$

18. (a): We have, $f(x)=x^2+\frac{1}{x^2}=\left(x-\frac{1}{x}\right)^2+2$

And $g(x)=x-\frac{1}{x}$ for $x\in R-\{-1,0,1\}$

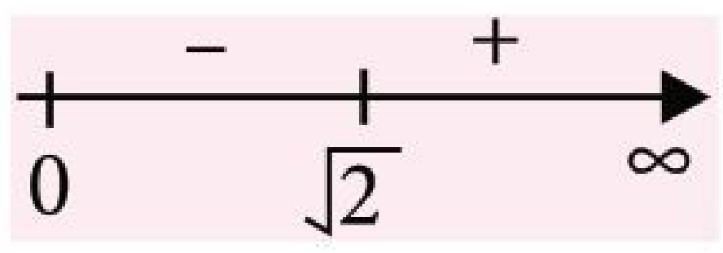
$$\therefore\quad h(x)=\frac{\left(x-\frac{1}{x}\right)^2+2}{x-\frac{1}{x}}=\left(x-\frac{1}{x}\right)+\frac{2}{\left(x-\frac{1}{x}\right)}$$

Put $x-\frac{1}{x}=t$

$\therefore\quad H(t)=t+\frac{2}{t}$ for $t\in(-\infty,\infty)-\{0\}$

Consider $H(t)$ on the interval $(0,\infty)$

$H(t)=t+\frac{2}{t}$; $H'(t)=1-\frac{2}{t^2}$



So, $H(t)$ is decreasing on $(0,\sqrt{2})$ and increasing on $(\sqrt{2},\infty)$. Thus $H(t)$ has a local minimum at $t=\sqrt{2}$.

$\therefore\quad H(\sqrt{2})=2\sqrt{2}$ is the local minimum value of the function at $\sqrt{2}$.

Remark : Observe that

$$t+\frac{2}{t}\geq 2\sqrt{2}$$

thereby again contribute that $2\sqrt{2}$ is a local minimum.

19. (c) : Let $I=\int\frac{\sin^2 x\cos^2 x}{(\sin^2 x+\cos^2 x)^2(\sin^3 x+\cos^3 x)^2}dx$

$$=\int\frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}dx=\int\frac{\tan^2 x\sec^2 x}{(1+\tan^3 x)^2}dx$$

Put 1 + $\tan^3 x = t$ so that $3\tan^2 x \sec^2 x\, dx = dt$

$$\therefore \quad I = \frac{1}{3}\int \frac{dt}{t^2} = -\frac{1}{3}\cdot\frac{1}{t} + C = -\frac{1}{3}\cdot\frac{1}{(1+\tan^3 x)} + C$$

20. (c)

21. (d): $\lim\limits_{x\to -1}\dfrac{f(x)}{(x+1)^3} = 1$

$\Rightarrow \quad f(-1) = f'(-1) = f''(-1) = 0, f'''(-1) = 6$

$\therefore \quad f(x) = A(x+1)^4 + (x+1)^3$

$\quad f'''(x) = 24A(x+1) + 6$

$f'''(0) = 0 \Rightarrow 24A + 6 = 0,\ A = -\dfrac{1}{4}$

$\therefore \quad f(x) = (x+1)^3 - \dfrac{1}{4}(x+1)^4$

$f'(x) = 3(x+1)^2 - (x+1)^3$

$f''(x) = 6(x+1) - 3(x+1)^2$

$f'(x) = 0 \Rightarrow (x+1)^2(2-x) = 0$

$\Rightarrow \quad x = -1, x = 2$ and $f''(2) = -9$

The maximum value is $f(2) = 27 - \dfrac{81}{4} = \dfrac{27}{4}$

22. (b): $f(0) = 1 - \dfrac{1}{4} = \dfrac{3}{4}, f'(0) = 2$

Subnormal at $x = 0$ is $f(0)\, f'(0) = \dfrac{3}{4} \times 2 = \dfrac{3}{2}$

23. (c) : $f''(x) = 3(x+1)(2-x-1) = 3(x+1)(1-x)$

$f''(x) = 0 \Rightarrow x = 1, -1$

24. (b): Since, the point of intersection of perpendicular tangents is (1, 1).

$\therefore$ (1, 1) is centre of the rectangular hyperbola.

$\therefore$ Equation of asymptotes are $(x-1) = 0$ and $(y-1) = 0$.

So the joint equation of asymptotes is $(x-1)(y-1) = 0$.

25. (d): We know that joint equation of asymptotes and hyperbola differ by a constant only.

Let equation of hyperbola is $(x-1)(y-1) + \lambda = 0$

Since it passes through the orthocentre (3, 2)

$\therefore \quad \lambda + 2 = 0$

$\therefore$ Equation of hyperbola is $(x-1)(y-1) = 2$.

26. (a) : We have, $(\vec{a}\times\vec{b})\times\vec{c} = \vec{a}\times(\vec{b}\times\vec{c})$

$\Rightarrow \quad (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

$\Rightarrow \quad \vec{a}$ and $\vec{c}$ are parallel.

$\therefore \quad \vec{c}$ may be equal to $4\hat{i} + 2\hat{j} + 4\hat{k}$

27. (d): We have, $\vec{b}\times\vec{c} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 4 & 2 & 4\end{vmatrix} = -6\hat{i} + 6\hat{k}$

$$\therefore \quad [\vec{a}\ \ \vec{b}\ \ \vec{b}\times\vec{c}] = \begin{vmatrix}2 & 1 & 2\\ 1 & -1 & 1\\ -6 & 0 & 6\end{vmatrix} = -36$$

$\therefore$ Volume of parallelepiped whose adjacent sides are $\vec{a}, \vec{b}, \vec{b}\times\vec{c}$ is 36 cubic units.

28. (a) : Vector along the bisectors is

$$\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}} + \frac{4\hat{i}+2\hat{j}+4\hat{k}}{6}$$

$$= \frac{1}{3}((2+\sqrt{3})\hat{i} + (1-\sqrt{3})\hat{j} + (2+\sqrt{3})\hat{k})$$

$\therefore$ Vector is $(2+\sqrt{3})\hat{i} + (1-\sqrt{3})\hat{j} + (2+\sqrt{3})\hat{k}$

29. (A) → (R); (B) → (P); (C) → (Q); (D) → (S)

30. (A) → (R); (B) → (P); (C) → (Q)

(A) $\displaystyle\int x^2\cdot\frac{1}{1+x^2}dx = x - \tan^{-1}x + c$

$\Rightarrow f(x) = \tan^{-1}x \Rightarrow f(1) = \tan^{-1}1 = \pi/4$

(B) $\displaystyle\int x^2 e^{2x}dx = \frac{1}{2}x^2\cdot e^{2x} - \int 2x\cdot\frac{1}{2}e^{2x}dx$

$$= \frac{1}{2}x^2e^{2x} - \left[x\cdot\frac{1}{2}e^{2x} - \frac{1}{4}e^{2x}\right] = \left(\frac{x^2}{2} - \frac{x}{2} + \frac{1}{4}\right)e^{2x} + c$$

(C) $\displaystyle\int\frac{(x^4+1)}{x(x^2+1)^2}dx = \int\frac{\left((x^2+1)^2 - 2x^2\right)dx}{x(x^2+1)^2}$

$$= \int\frac{1}{x}dx - 2\int\frac{xdx}{(x^2+1)^2} = \log|x| + \frac{1}{x^2+1} + c$$

31. (A) → (P, S); (B) → (Q); (C) → (R); (D) → (P, S)

32. (1): Let $I = \displaystyle\int\frac{(\sqrt{x})^5}{(\sqrt{x})^7 + x^6}dx$

$$= \int\frac{dx}{(\sqrt{x})^2 + (\sqrt{x})^7} = \int\frac{1}{(\sqrt{x})^7\left(1+\frac{1}{(\sqrt{x})^5}\right)}dx$$

Let $1 + \dfrac{1}{(\sqrt{x})^5} = t$. Then $-\dfrac{5}{2}x^{-7/2}\,dx = dt$

$$\therefore \quad I = -\frac{2}{5}\int\frac{dt}{t} = -\frac{2}{5}\log|t| + C = \frac{2}{5}\log\left(\frac{x^{5/2}}{1+x^{5/2}}\right) + C$$

On comparing $a\log\left(\dfrac{x^\lambda}{x^\lambda+1}\right) + C$ with

$\dfrac{2}{5}\log\left(\dfrac{x^{5/2}}{1+x^{5/2}}\right) + C$, we get $a = \dfrac{2}{5}$ and $\lambda = \dfrac{5}{2}$

$\therefore \quad a\lambda = 1$.

33. (8) : Since $\lim\limits_{x\to\infty}\dfrac{x^2 f(x)}{2x^5+3}=6$, $f(x)$ must be of degree 3.

Also from $f(1) = 3$, $f(3) = 7$ and $f(7) = 15$, we get,
$f(x) = \lambda(x - 1)(x - 3)(x - 7) + 2x + 1$

$$\therefore \quad \lim_{x\to\infty}\frac{x^2\cdot(\lambda(x-1)(x-3)(x-7)+2x+1)}{2x^5+3}=6$$

$$\Rightarrow \quad \frac{\lambda}{2}=6 \;\Rightarrow \lambda = 12$$

$\therefore \quad f(x) = 12(x - 1)(x - 3)(x - 7) + 2x + 1$

$\therefore \quad f(0) = -12 \times 21 + 1 = -251$

$$\left[\frac{|f(0)|}{30}\right]=\left[\frac{251}{30}\right]=[8.366]=8$$

34. (32) : We have, $a_n = \tan^{-1}\left(\dfrac{3}{9n^2+3n-1}\right)$

$$=\tan^{-1}\left(\frac{3}{1+(3n+2)(3n-1)}\right)$$

$$=\tan^{-1}\left(\frac{(3n+2)-(3n-1)}{1+(3n+2)(3n-1)}\right)$$

$= \tan^{-1}(3n + 2) - \tan^{-1}(3n - 1)$

$\therefore$ Sum of first 10 terms

$$=\sum_{r=1}^{10}a_r=\sum_{r=1}^{10}[\tan^{-1}(3r+2)-\tan^{-1}(3r-1)]$$

$= (\tan^{-1}5 - \tan^{-1}2) + (\tan^{-1}8 - \tan^{-1}5) + \ldots$
$+ (\tan^{-1}32 - \tan^{-1}29)$

$$=\tan^{-1}32-\tan^{-1}2=\tan^{-1}\left(\frac{32-2}{1+32\cdot 2}\right)$$

$$=\tan^{-1}\left(\frac{30}{65}\right)=\tan^{-1}\left(\frac{6}{13}\right)=\cot^{-1}\left(\frac{13}{6}\right)=\cot^{-1}\left(\frac{m}{n}\right)$$

[Given]

$\Rightarrow \quad m = 13$ and $n = 6$

Hence, $(2m + n) = 32$

35. (1) : We have, $\dfrac{dy}{dx}=-\dfrac{\sqrt{(x^2-1)(y^2-1)}}{xy}$

$$\Rightarrow \quad \int\frac{y}{\sqrt{y^2-1}}dy=-\int\frac{\sqrt{x^2-1}}{x}dx=-\int\frac{x^2-1}{x\sqrt{x^2-1}}dx$$

$$\Rightarrow \quad \sqrt{y^2-1}=-\int\frac{x}{\sqrt{x^2-1}}dx+\int\frac{1}{x\sqrt{x^2-1}}dx$$

$$\Rightarrow \quad \sqrt{y^2-1}=-\sqrt{x^2-1}+\sec^{-1}x+c$$

Curve passes through the point (1, 1). Then the value of $c = 0$.

Hence, the curve is $\sqrt{y^2-1}=-\sqrt{x^2-1}+\sec^{-1}(x)$

Curve passes through $(\sqrt{2},k)$

$$\therefore \quad \sqrt{k^2-1}=-1+\frac{\pi}{4}\Rightarrow k=1 \quad [\because \sqrt{k^2-1}\text{ can't be}<0]$$

36. (11) : $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$

$\det P = 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) = 2\alpha - 6$

Also, $\det(\operatorname{adj} A) = (\det A)^2$

$\Rightarrow \quad 2\alpha - 6 = 16 \Rightarrow 2\alpha = 22 \quad \therefore \quad \alpha = 11$

37. (879) : Number of ways in which one or more balls can be selected from 10 white, 9 green, 7 black balls is $= (10 + 1)(9 + 1)(7 + 1) - 1 = 880 - 1 = 879$ ways

38. (1.69) : $\cos(\alpha + \beta) = 4/5$ gives $\tan(\alpha + \beta) = 3/4$

Also $\sin(\alpha - \beta) = 5/13$ gives $\tan(\alpha - \beta) = 5/12$

$\tan(2\alpha) = \tan(\alpha + \beta + \alpha - \beta)$

$$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}=\frac{56}{33}=1.69$$

39. (7) : $\lim\limits_{x\to1}\dfrac{F(x)}{G(x)}=\lim\limits_{x\to1}\dfrac{F'(x)}{G'(x)}=\lim\limits_{x\to1}\dfrac{f(x)}{x|f(f(x))|}=\dfrac{1}{14}$

$$=\frac{f(1)}{\left|f\left(\frac{1}{2}\right)\right|}=\frac{1}{14}$$

Thus $\left|f\left(\frac{1}{2}\right)\right|=7$. But $f\left(\frac{1}{2}\right)\neq-7$ as $f(x)$ vanishes exactly at one point.

40. (10.1) : The numbers are $1, 1 + d, 1 + 2d, \ldots, 1 + 100d$.

The numbers are in A.P.

Then mean = 51st term = $1 + 50d$ = (say)

Mean deviation (M.D.) $=\dfrac{1}{n}\sum\limits_{i=1}^{101}|x_i-\bar{x}|$

$$=\frac{1}{101}[50d+49d+48d+\ldots.+d+0+d+2d+\ldots+50d]$$

$$=\frac{1}{101}\cdot 2d(1+2+\ldots.+50)=\frac{1}{101}\cdot 2d\cdot\frac{50\cdot 51}{2}=\frac{50\cdot 51}{101}d$$

But M.D. = 255 (given)

$$\Rightarrow \quad \frac{50\cdot 51}{101}d=255 \Rightarrow d=\frac{101\times 255}{50\times 51}=\frac{101\times 255}{2550}=10.1$$

BRUSH UP for JEE Main & Advanced 2022

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Permutations and Combinations & Binomial Theorem

Single Correct Answer Type

1. The coefficient of $a^7b^{10}c^3$ in the expansion of $(ab + bc + ca)^{10}$ is

(a) 120 (b) 110
(c) 90 (d) None of these

2. The value of $\sum_{r=0}^{n} \frac{{}^{n+1}C_r}{{}^{n+1}C_r + {}^{n+1}C_{r+1}}$ is

(a) $n + 2$ (b) $\frac{n+3}{2}$
(c) $\frac{n+1}{2}$ (d) $\frac{n+2}{2}$

3. The number of 9 digit numbers formed by using the digits 4, 4, 5, 5, 7, 7, 6, 6, 6, such that odd digits occupy even places is

(a) 120 (b) 90 (c) 60 (d) 36

4. A candidate goes for an examination in which there are 5 paper with a maximum of 12 marks for each paper, the number of ways of obtaining 32 marks on the whole is

(a) 16730 (b) 12530
(c) 10528 (d) None of these

5. The number of non-negative integral solutions of equation $4x + y + z = 32$, is

(a) 11×3^2 (b) $7^2 \times 11$
(c) $5^2 \times 17$ (d) $3^2 \times 17$

6. The vertices of a regular polygon of 12 sides are joined to form triangles. The number of triangles, which do not have their sides as the sides of the polygon, is

(a) 96 (b) 108 (c) 220 (d) 112

7. In the expansion of $(ax + b)^{2019}$, if the coefficient of x^3 and x^4 are equal, then $\frac{b}{a}$ is equal to

(a) 505 (b) 504
(c) 502 (d) 672

8. If $\frac{{}^nC_r + 3\,{}^nC_{r+1} + 3\,{}^nC_{r+2} + {}^nC_{r+3}}{{}^nC_r + 4\,{}^nC_{r+1} + 6\,{}^nC_{r+2} + 4\,{}^nC_{r+3} + {}^nC_{r+4}} = \frac{r+k}{n+k}$, then find the value of k.

(a) 1 (b) 2 (c) 3 (d) 4

More Than One Correct Answer Type

9. The value of $\sum_{r=0}^{n} (-2)^r \cdot \frac{{}^nC_r}{{}^{r+2}C_r}$ is

(a) $\frac{1}{n+2}$, if n is odd (b) $\frac{1}{n+2}$, if n is even
(c) $\frac{1}{n+1}$, if n is even (d) $\frac{1}{n+1}$, if n is odd

10. The number of positive solutions of $x + y + z + u + v \le 15$ and $x + y + z = 7$, is divisible by

(a) 2 (b) 3
(c) 5 (d) 7

11. In a class, there are 20 students. The following prizes are to be awarded to the students of the class.

(i) First and second in Mathematics
(ii) First and second in Physics.
(iii) First in Mathematics and first in Biology

If N denotes the number of ways in which prizes can be awarded, then which of the following is true?

(a) $1900 | N$
(b) $2500 | N$
(c) $36100 | N$
(d) N is divisible by three distinct prime numbers.

12. $(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ and let $\sum_{r=0}^{n}(r^2 - 3r + 1)C_r = 2^{n-2}f(n)$. If roots of $f(x) = 0$ are α and β, then
(a) $3\alpha + 4\beta = 16$ or 19
(b) $\alpha^2\beta + \beta^2\alpha = 20$
(c) $f(x + 1) = 0$ is the equation, whose roots are 1 less than the roots of original equation.
(d) $(\alpha + 1)$ and $(\beta + 1)$ are roots of the equation $f(x - 1) = 0$.

13. If $(1 + x^2)^2 (1 + x)^n = \sum_{k=0}^{n+4} a_k x^k$, where a_1, a_2, a_3 are in A.P., then value of n is
(a) 2 (b) 3 (c) 4 (d) 5

14. If $(1 + x + 2x^2)^{20} = \sum_{r=0}^{40} a_r x^r$, then
(a) $\sum_{r=0}^{19} a_{2r} = 2^{19}(2^{20} - 1)$ (b) $a_{40} = 2^{20}$
(c) $a_{39} = 5 \cdot 2^{21}$ (d) $\sum_{r=0}^{40} a_r (-1)^r = 2^{20}$

15. If $x = 100$, then
(a) $E_2(x!) = 3$ less than x
(b) The number of divisor of $E_5(x!) = 8$
(c) $E_{10}(x!) = 24$
(d) $E_{10}(x!)$ is minimum of $E_2(x!)$ and $E_5(x!)$

Comprehension Type

Paragraph for Q. No. 16 to 18

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$
On the basis of above information answer the following questions.

16. The number of six digit numbers that can be formed by using the members of set A, so that digit do not repeated, is
(a) ${}^7P_6$ (b) ${}^6P_6$ (c) $6!\,{}^7P_6$ (d) $7!\,{}^6P_6$

17. The number of six digit numbers formed by using the elements of set A, without repetition and terminal digits are even, is
(a) 244 (b) 720
(c) 144 (d) 120

18. The number of four digit numbers formed by using the digits of set A with repetition and terminal digits are odd, is
(a) $(4!)^2 (7!)^2$ (b) 2401
(c) 588 (d) 784

Matrix Match Type

19. Match the following :

Column-I		Column-II	
A.	Number of triangles joining the vertices of the polygon having 35 diagonals, is	P.	1296
B.	Maximum number of point of intersection of six circles in the plane, is	Q.	57
C.	Number of straight lines joining any two of 12 points of which 5 are collinear, is	R.	120
D.	Number of rectangles in a 8 × 8 chess board, is	S.	30

Numerical Answer Type

20. The difference of maximum and minimum value of r for which ${}^{30}C_r\,{}^{20}C_0 + {}^{30}C_{r-1}\,{}^{20}C_1 + {}^{30}C_{r-2}\,{}^{20}C_2 + ... + {}^{30}C_0\,{}^{20}C_r$ is maximum and minimum is ________.

21. If $27^{10} + 7^{51}$ is divided by 10, then the remainder is ________.

22. A five digit number divisible by 3 is to be formed by using the numerals 0, 1, 2, 4, 5 and 6 without repetition. If the total number of ways of doing so is n^3, then value of $n!$ is ________.

23. If the number of ways in which 6 boys and 5 girls can be arranged on a circular table such that no two girls sit together and two particular boys are always together is $10n^2$, then sum of divisors of n is ________.

24. If the letters of the word 'SCHOOL' are arranged in all possible ways as in a dictionary, then rank of the word 'SCHOOL' from the last is __________.

25. If L_1 be the number of ways in which four letters out of the letters of the word EXAMINATION can be selected and L_2 be the number of ways in which four letters out of the letter of the word MATHEMATICS can be selected, then $L_1 + L_2$ equals _________.

SOLUTIONS

1. (a) : General term of $(ab + bc + ca)^{10}$ is

$$= \frac{10!}{p!q!r!}(ab)^p (bc)^q (ca)^r$$

$$= \frac{10!}{p!q!r!}a^{p+r} \cdot b^{p+q} \cdot c^{q+r} \quad ...(i)$$

According to question, let $p + r = 7$ ...(ii)

$p + q = 10$ …(iii)

and $q + r = 3$ …(iv)

Adding (ii), (iii) and (iv), we get

$p + q + r = 10$ …(v)

Solving above equations, we get $p = 7$, $q = 3$ and $r = 0$

From (i), coefficient of $a^7b^{10}c^3 = \frac{10!}{7!3!0!} = \frac{10\cdot 9\cdot 8}{6} = 120$

2. (c) : Since, ${}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1}$

$$= \frac{n+2}{r+1}\cdot {}^{n+1}C_r \qquad \left(\because\ {}^{n}C_r = \frac{n}{r}\cdot {}^{n-1}C_{r-1}\right)$$

$$\therefore \quad \sum_{r=0}^{n} \frac{{}^{n+1}C_r}{{}^{n+1}C_r + {}^{n+1}C_{r+1}} = \sum_{r=0}^{n} \frac{r+1}{n+2}$$

$$= \frac{1}{(n+2)}\sum_{r=0}^{n}(r+1) = \frac{1}{(n+2)}[1 + 2 + 3 + \dots + (n + 1)]$$

$$= \frac{(n+1)(n+2)}{(n+2)\cdot 2} = \frac{n+1}{2}$$

3. (c) : Number of odd digits (5′s = 2, 7′s = 2) = 4

= Number of even places (2nd, 4th 6th, 8th places)

$\therefore$ Number of ways placing 5, 5, 7, 7 in even places

$$= \frac{4!}{2!2!} = 6 \text{ ways}$$

Similarly, number of ways placing even digits

(4, 4, 6, 6, 6) on 5 odd places $= \frac{5!}{2!3!}$ = 10 ways

$\therefore$ Number of 9 digit numbers in which odd digits occupies even places = 6 × 10 = 60

4. (a) : Maximum marks of each paper is 12, which means the candidate can obtain 0 to 12 marks in each paper and there are 5 papers in which he needed 32 marks in total.

$\therefore$ Using multinomial theorem, the required number of ways of getting 32 marks

= Coefficient of x^{32} in $(x^0 + x^1 + x^2 + \dots + x^{12})^5$

= Coefficient of x^{32} in $(1 - x^{13})^5 (1 - x)^{-5}$

$$= \text{Coefficient of } x^{32} \text{ in } \left[1-\binom{5}{1}x^{13}+\binom{5}{2}x^{26}+\dots\right]\times$$

$$\left[1+\binom{5}{1}x+\binom{6}{2}x^2+\dots+\binom{10}{6}x^6+\dots+\binom{23}{19}x^{19} + \dots + \binom{36}{32}x^{32}+\dots\right]$$

$$= \binom{36}{32} - \binom{5}{1}\binom{23}{19} + \binom{5}{2}\binom{10}{6}$$

= 58905 − (5 × 8855) + (10 × 210) = 16730

5. (d) : $\because$ $4x + y + z = 32$ and x, y, z assumes the non negative integral values.

$\therefore$ $x \ge 0$, $y \ge 0$, and $z \ge 0$

Let $x = k$

$\therefore$ $y + z = 32 - 4k$ …(i)

$\Rightarrow$ $32 \ge 32 - 4k \ge 0$ $(\because\ x \ge 0)$

$\Rightarrow$ $32 \ge 32 - 4k$ and $32 - 4k \ge 0$

$\Rightarrow$ $k \ge 0$ and $k \le 8 \Rightarrow 0 \le k \le 8$

$\therefore$ Number of non-negative solutions of equation (i) is

$= {}^{32-4k+2-1}C_{2-1}$

$= {}^{33-4k}C_1 = 33 - 4k$

$\therefore$ Total number of solutions of equation

$4x + y + z = 32$

$$= \sum_{k=0}^{8}(33-4k) = 33\times\sum_{k=0}^{8}1 - 4\sum_{k=0}^{8}k = 33\times 9 - 4\cdot\frac{8\times 9}{2}$$

$= 33 \times 9 - 16 \times 9 = 9 \times 17 = 17 \times 3^2$

6. (d) : Total number of triangles $= {}^{12}C_3 = 220$

The number of triangles with two sides common with polygon = 12

The number of triangles with one side common with polygon = 8 × 12 = 96

$\therefore$ Required number of triangles = 220 − (12 + 96)
= 220 − 108 = 112

7. (b) : $\because$ $(ax + b)^{2019} = (b + ax)^{2019}$

$= b^{2019} + {}^{2019}C_1\, b^{2018}(ax) + {}^{2019}C_2\, b^{2017}(ax)^2 + {}^{2019}C_3\, b^{2016}(ax)^3 + {}^{2019}C_4\, b^{2015}(ax)^4 + \dots$

Now, coefficient of x^3 and x^4 are equal

$\therefore$ ${}^{2019}C_3\, b^{2016}\, a^3 = {}^{2019}C_4\, b^{2015}\, a^4$

$$\Rightarrow \frac{b}{a} = \frac{{}^{2019}C_4}{{}^{2019}C_3} = \frac{2016}{4} = 504$$

8. (d) : $$\frac{\left[\begin{array}{c}({}^nC_r + {}^nC_{r+1}) + 2({}^nC_{r+1} + {}^nC_{r+2}) \\ +({}^nC_{r+2} + {}^nC_{r+3})\end{array}\right]}{\left[\begin{array}{c}({}^nC_r + {}^nC_{r+1}) + 3({}^nC_{r+1} + 2\,{}^nC_{r+2} + {}^nC_{r+3}) \\ +({}^nC_{r+3} + {}^nC_{r+4})\end{array}\right]}$$

$$= \frac{{}^{n+1}C_{r+1} + 2({}^{n+1}C_{r+2}) + {}^{n+1}C_{r+3}}{{}^{n+1}C_{r+1} + 3({}^{n+1}C_{r+2} + {}^{n+1}C_{r+3}) + {}^{n+1}C_{r+4}}$$

$$= \frac{({}^{n+1}C_{r+1} + {}^{n+1}C_{r+2}) + ({}^{n+1}C_{r+2} + {}^{n+1}C_{r+3})}{({}^{n+1}C_{r+1} + {}^{n+1}C_{r+2}) + 2({}^{n+2}C_{r+3}) + {}^{n+2}C_{r+4}}$$

$$= \frac{{}^{n+2}C_{r+2} + {}^{n+2}C_{r+3}}{({}^{n+2}C_{r+2} + {}^{n+2}C_{r+3}) + ({}^{n+2}C_{r+3} + {}^{n+2}C_{r+4})}$$

$$= \frac{{}^{n+3}C_{r+3}}{{}^{n+4}C_{r+4}} = \frac{{}^{n+3}C_{r+3}}{{}^{n+3}C_{r+3}}\frac{(r+4)}{(n+4)} = \frac{r+4}{n+4}$$

$$\therefore \quad \frac{r+4}{n+4}=\frac{r+k}{n+k} \Rightarrow k=4$$

9. (a,c) : We have, $\sum\limits_{r=0}^{n}(-2)^r \cdot \frac{{}^nC_r}{{}^{r+2}C_r}$

$$=\sum_{r=0}^{n}\left[(-2)^r \cdot \frac{r!2!}{(r+2)!}\,{}^nC_r\right] = \sum_{r=0}^{n}\left[(-2)^r \frac{(2)\,{}^nC_r}{(r+2)(r+1)}\right]$$

$$=\frac{1}{2(n+1)(n+2)}\sum_{r=0}^{n}\left[(-2)^{r+2}\frac{(n+2)(n+1)}{(r+2)(r+1)}\,{}^nC_r\right]$$

$$=\frac{1}{2(n+1)(n+2)}\sum_{r=0}^{n}\left[(-2)^{r+2}\cdot{}^{n+2}C_{r+2}\right]$$

$$=\frac{1}{2(n+1)(n+2)}\left[{}^{n+2}C_2(-2)^2+{}^{n+2}C_3(-2)^3+\ldots.+{}^{n+2}C_{n+2}(-2)^{n+2}\right]$$

$$=\frac{1}{2(n+1)(n+2)}\left[{}^{n+2}C_0(-2)^0+{}^{n+2}C_1(-2)^1+{}^{n+2}C_2(-2)^2+\ldots.+{}^{n+2}C_{n+2}(-2)^{n+2}-{}^{n+2}C_0(-2)^0-{}^{n+2}C_1(-2)^1\right]$$

$$=\frac{1}{2(n+1)(n+2)}[(1-2)^{n+2}-(1+(n+2)(-2))]$$

$$=\frac{1}{2(n+1)(n+2)}\left[(-1)^{n+2}+2(n+2)-1\right]$$

$$=\begin{cases}\dfrac{1}{2(n+1)(n+2)}[1+2(n+2)-1], \text{ if } n \text{ is even}\\[2ex] \dfrac{1}{2(n+1)(n+2)}[-1+2(n+2)-1], \text{ if } n \text{ is odd}\end{cases}$$

$$=\begin{cases}\dfrac{2(n+2)}{2(n+1)(n+2)}, \text{ if } n \text{ is even}\\[2ex] \dfrac{2(n+1)}{2(n+1)(n+2)}, \text{ if } n \text{ is odd}\end{cases} = \begin{cases}\dfrac{1}{n+1}, \text{ if } n \text{ is even}\\[2ex] \dfrac{1}{n+2}, \text{ if } n \text{ is odd}\end{cases}$$

So, option (a) and (c) are correct.

10. (a,b,c,d) : We have, $x + y + z = 7$ ...(i)

Number of solutions of (i) $= {}^{7-1}C_{3-1} = {}^6C_2 = 15$

Again, $x + y + z + u + v \le 15 \Rightarrow u + v \le 8$ ...(ii)

Number of solutions of (ii)

$$=\binom{7}{1}+\binom{6}{1}+\ldots+\binom{1}{1}=\frac{7\times 8}{2}=28$$

$\therefore$ Total number of solutions of (i) and (ii)

$= 15 \times 28 = 2^2 \cdot 3^1 \cdot 5^1 \cdot 7^1$, which is divisible by 2, 3, 5 and 7.

$\therefore$ All options are correct.

11. (a,b,c,d) : Total number of students = 20

Number of ways of awarding first and second prizes in Mathematics $= {}^{20}P_2$

Number of ways of awarding first and second prizes in Physics $= {}^{20}P_2$

Number of ways of awarding first prize in Mathematics and first prize in Biology $= 20 \times 20$

$\therefore \quad N = {}^{20}P_2 \times {}^{20}P_2 \times 20 \times 20$

$= (20)^4 \times (19)^2 = 2^8 \times 5^4 \times 19^2$ which is divisible by three distinct primes *i.e.*, 2, 5, 19 (option (d) is correct)

Now, $1900 = 100 \times 19 = 2^2 \times 5^2 \times 19$

$2500 = 100 \times 25 = 2^2 \times 5^4$

$36100 = 100 \times 19^2 = 2^2 \times 5^2 \times 19^2$

$\therefore \quad N = 2^8 \times 5^4 \times 19^2$ is divisible by each of 1900, 2500 and 36100 (options (a), (b), (c) are correct)

12. (a,b,c,d) : $\because \sum\limits_{r=0}^{n}(r^2-3r+1)C_r = 2^{n-2}f(n)$

$$\Rightarrow \sum_{r=0}^{n}[r(r-1)-2r+1]C_r = 2^{n-2}f(n)$$

$$\Rightarrow \sum_{r=0}^{n}r(r-1)\,C_r - 2\sum_{r=0}^{n}r\,C_r + \sum_{r=0}^{n}C_r = 2^{n-2}f(n)$$

$\Rightarrow \quad n(n-1)\,2^{n-2} - 2n\,2^{n-1} + 2^n = 2^{n-2}f(n)$

$\Rightarrow \quad 2^{n-2}[n(n-1) - 4n + 4] = 2^{n-2}f(n)$

$\Rightarrow \quad f(n) = n^2 - 5n + 4 \Rightarrow f(x) = x^2 - 5x + 4$

$\Rightarrow \quad f(x) = (x-1)(x-4)$

$\Rightarrow \quad \alpha = 4, \beta = 1$ or $\alpha = 1, \beta = 4$ $\quad [\because f(x) = 0]$

$\therefore \quad 3\alpha + 4\beta = 12 + 4 = 16$ or $3 + 16 = 19$

(option (a) is correct)

Again, $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha + \beta) = 4(5) = 20$

(option (b) is correct)

Also, $f(x) = x^2 - 5x + 4$

$\therefore \quad f(x+1) = x^2 - 3x = x(x-3)$

$\therefore$ Roots of $f(x+1) = 0$ and 3 and each root is 1 less than the roots of $f(x) = 0$.

(option (c) is correct)

Now, $f(x-1) = 0 \Rightarrow (x-1)^2 - 5(x-1) + 4 = 0$

$\Rightarrow \quad x^2 - 7x + 10 = 0 \Rightarrow (x-2)(x-5) = 0$

$\Rightarrow \quad x = 2, 5 = 1 + 1, 1 + 4 = 1 + \alpha, 1 + \beta$

i.e., $\alpha + 1, \beta + 1$ are roots of $f(x-1) = 0$

(option (d) is correct)

13. (a,b,c) : $\because \quad (1+x^2)^2(1+x)^n = \sum\limits_{k=0}^{n+4}a_k x^k$

$$\Rightarrow \quad (1 + 2x^2 + x^4)\left(1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\ldots.\right)$$

$= a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_{n+4}x^{n+4}$

Comparing the coefficients of x, x^2 and x^3, we get

$$a_1 = n,\ a_2 = \frac{n(n-1)}{2}+2=\frac{n^2-n+4}{2},$$

$$a_3=\frac{n(n-1)(n-2)}{6}+2n=\frac{n^3-3n^2+14n}{6}$$

Given, a_1, a_2, a_3 are in A.P.

$\Rightarrow\quad \frac{n^3-3n^2+14n}{6}+n=\frac{2(n^2-n+4)}{2}$

$\Rightarrow\quad n^3-9n^2+26n-24=0$

$\Rightarrow\quad (n-2)(n-3)(n-4)=0\Rightarrow n=2,3,4$

$\therefore$ Options (a), (b) and (c) are correct.

14. (a,b,c,d) : $\because\ (1+x+2x^2)^{20}=\sum_{r=0}^{40}a_rx^r$

$= a_0+a_1x+a_2x^2+\ldots.+a_{40}x^{40}$ ...(i)

Putting $x = 1$ and $x = -1$ in (i), we get

$a_0+a_1+a_2+a_3+\ldots+a_{38}+a_{39}+a_{40}=4^{20}$ ...(ii)

$a_0-a_1+a_2-a_3+\ldots+a_{38}-a_{39}+a_{40}=2^{20}$ ...(iii)

Adding (ii) and (iii), we get

$2(a_0+a_2+a_4+\ldots+a_{38}+a_{40})=4^{20}+2^{20}=2^{20}(2^{20}+1)$

$\Rightarrow\quad a_0+a_2+a_4+\ldots+a_{38}+a_{40}=2^{19}(2^{20}+1)$...(iv)

Now, a_{40} = Coefficient of x^{40} in $[1+x(1+2x)]^{20}$

and for coefficient of x^{40}, we have,

$T_{r+1}={}^{20}C_rx^r(1+2x)^r$

Putting $r = 20$, we have $T_{21}={}^{20}C_{20}\,x^{20}\,(1+2x)^{20}$

$\Rightarrow\quad T_{21}=x^{20}\,[1+{}^{20}C_1(2x)^1+{}^{20}C_2(2x)^2+\ldots$

$+\,{}^{20}C_{19}\,(2x)^{19}+{}^{20}C_{20}(2x)^{20}]$ ...(v)

$\therefore$ Coefficient of x^{40} in $[1+x\,(1+2x)]^{20}$ is ${}^{20}C_{20}2^{20}$

$= 2^{20}$ (option (b) is correct)

Now, $\sum_{r=0}^{19}a_{2r}=a_0+a_2+a_4+\ldots.+a_{38}$

$=2^{19}(2^{20}+1)-a_{40}$ [Using (iv)]

$\Rightarrow\quad a_0+a_2+a_4+\ldots+a_{38}=2^{19}(2^{20}+1)-2^{20}$

$=2^{19}(2^{20}-1)$

(option (a) is correct)

Now, a_{39} = Coefficient of x^{39} in $[1+x(1+2x)]^{20}$

$={}^{20}C_{19}2^{19}$ [Using (v)]

$=20\cdot2^{19}=5\cdot2^{21}$ (option (c) is correct)

Again, $(1+x+2x^2)^{20}=\sum_{r=0}^{40}a_rx^r$

Putting $x = -1$

$\therefore\ (1-1+2(-1)^2)^{20}=\sum_{r=0}^{40}a_r(-1)^r$

$\therefore\ \sum_{r=0}^{40}a_r(-1)^r=2^{20}$ (option (d) is correct)

15. (a,b,c,d) : $E_2(x!)$ at $x = 100 = E_2(100!)$

$$=\left[\frac{100}{2}\right]+\left[\frac{100}{2^2}\right]+\left[\frac{100}{2^3}\right]+\left[\frac{100}{2^4}\right]+\left[\frac{100}{2^5}\right]+\left[\frac{100}{2^6}\right]$$

$= 50 + 25 + 12 + 6 + 3 + 1 = 97$

$= 100 - 3$ (option (a) is correct)

Again, $E_5(x!)=E_5(100!)=\left[\frac{100}{5}\right]+\left[\frac{100}{25}\right]$

$=20+4=24=2^3\cdot3^1$

$\therefore$ Number of divisors of $E_5(x!)=(3+1)(1+1)$

$=4\times2=8$

(option (b) is correct)

Now, $E_{10}(x!)=\min\,(97,24)=24$

(option (c) and (d) are correct)

16. (a) : we have, $A=\{1,2,3,4,5,6,7\}$

$\Rightarrow\quad n(A)=7$ (All distinct)

We need six digit numbers, when repetition of digits is not allowed.

$\therefore$ Required number of six digit numbers $={}^7P_6$ or $7!$

17. (b) : As repetition is not allowed but terminal digits are even, there are 3 even digits namely, 2, 4 and 6.

$$\text{Terminal}\ \boxed{3\text{ ways}}\ \ \underbrace{|\ \ |\ \ |\ \ |}_{{}^5P_4\text{ ways}=120}\ \ \boxed{2\text{ ways}}\ \text{Terminal}$$

Required six digit numbers $= 3\times120\times2 = 720$

18. (d) : There are 4 odd digits namely 1, 3, 5, 7 and two of them occupy the terminal places.

$\therefore$ Required 4 digit odd number $=4^2\times7^2$

$= 16\times49 = 784$

$$\boxed{4\text{ ways}}\ \ \underbrace{|\ \ |}_{n^r=7^2}\ \ \boxed{4\text{ ways}}$$

19. (A) → (R) ; (B) → (S) ; (C) → (Q) ; (D) → (P)

(A) We know that if polygon has n sides, then the total number of diagonals is ${}^nC_2-n$.

$\therefore\ {}^nC_2-n=35\Rightarrow\frac{n(n-3)}{2}=35$

$\Rightarrow\ n^2-3n-70=0\Rightarrow(n-10)(n+7)=0$

$\therefore\ n=10$

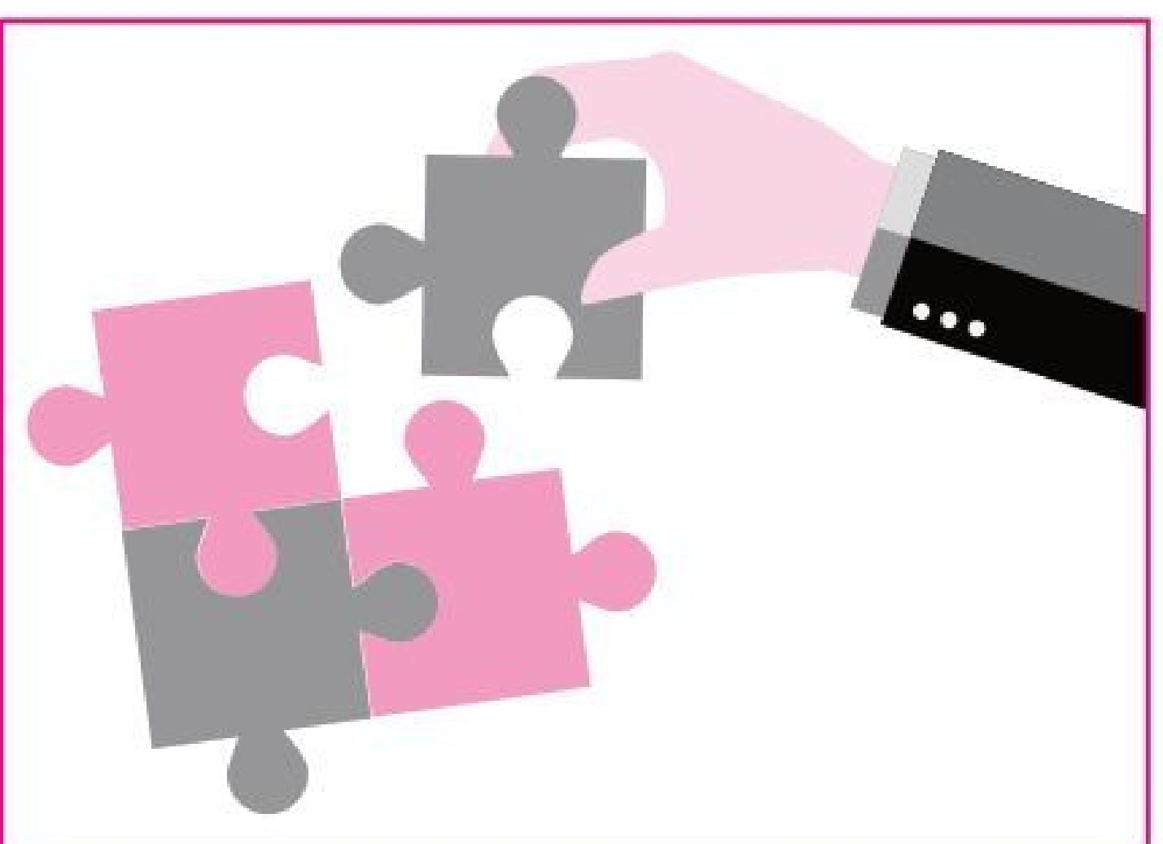
Now, with 10 vertices, number of triangles formed $={}^{10}C_3=120$

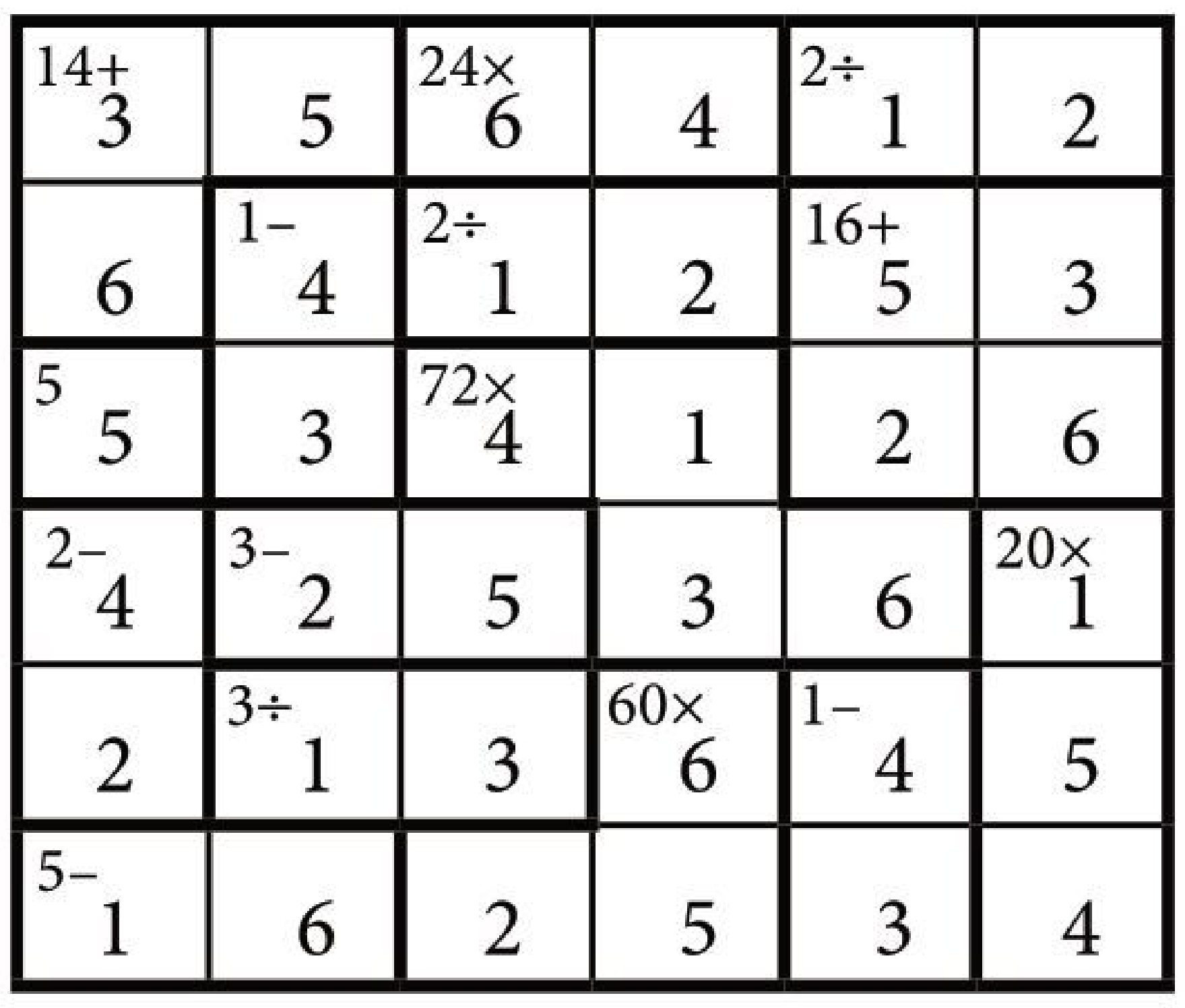
(B) Number of circles = 6 (Given)

$\therefore$ Maximum number of point of intersection $=2\times{}^6C_2=30$

PUZZLE CORNER

SOLUTION · DECEMBER 2021





14+ 3	5	24× 6	4	2÷ 1	2
6	1– 4	2÷ 1	2	16+ 5	3
5 5	3	72× 4	1	2	6
2– 4	3– 2	5	3	6	20× 1
2	3÷ 1	3	60× 6	1– 4	5
5– 1	6	2	5	3	4

(C) Number of straight lines with 12 points = ${}^{12}C_2 = 66$
Number of straight lines with 5 points = ${}^{5}C_2 = 10$
$\therefore$ Required number of straight lines
$= 66 - 10 + 1 = 57$

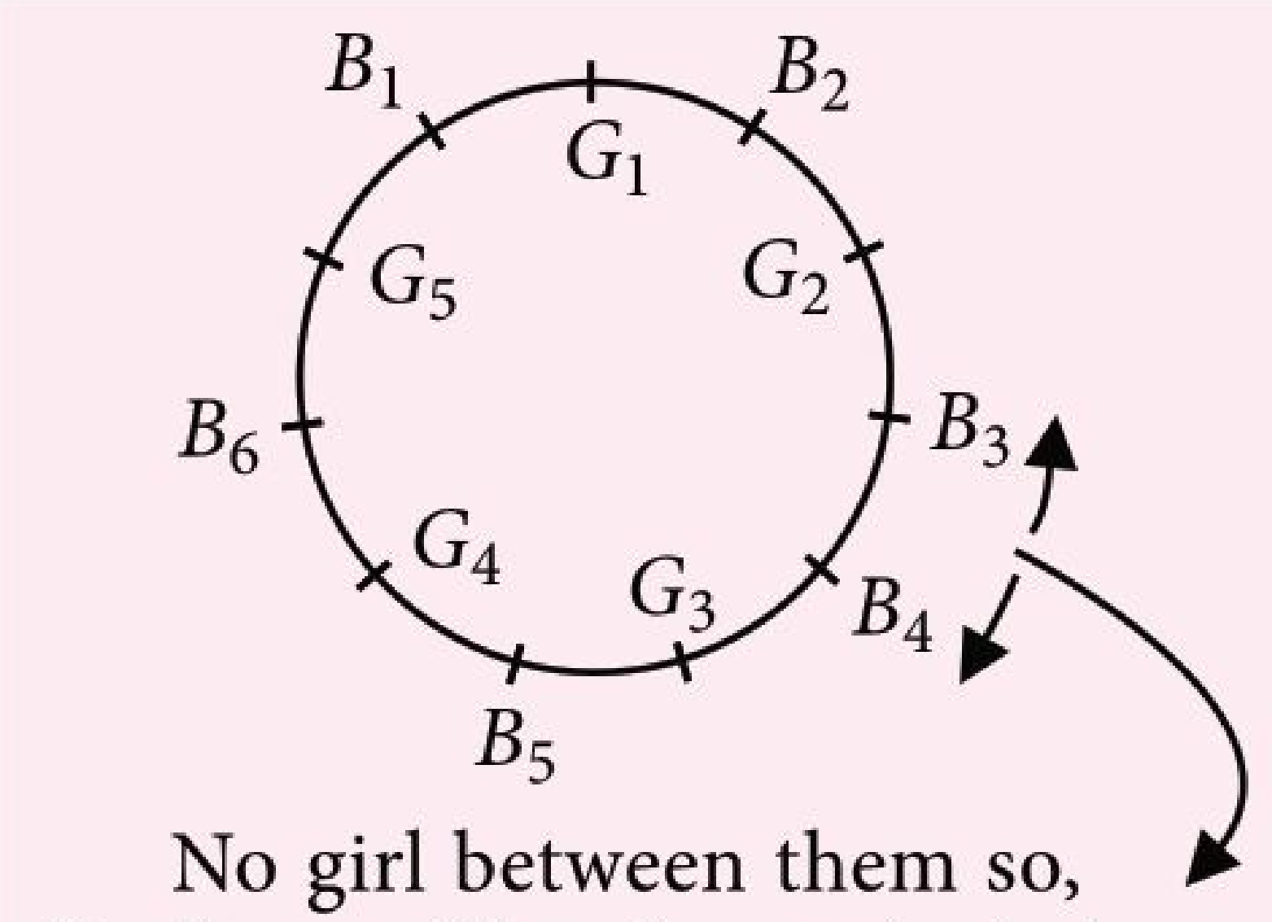
(D) A 8 × 8 chess board having 9 horizontal lines and 9 vertical lines.
$\therefore$ Required number of rectangles formed
$= {}^{9}C_2 \times {}^{9}C_2 = 1296$

20. (25) : We have, ${}^{30}C_r\,{}^{20}C_0 + {}^{30}C_{r-1}\,{}^{20}C_1 + \ldots + {}^{30}C_0\,{}^{20}C_r$ ...(i)
= Coefficient of x^r in $(1+x)^{30} \cdot (1+x)^{20}$
= Coefficient of x^r in $(1+x)^{50}$
$= {}^{50}C_r = {}^{n}C_r \Rightarrow n = 50$, which is even
$\therefore\ r = \frac{n}{2} = 25$ for maximum value of (i) and $r = 0$ for minimum value of (i).
$\therefore$ Required difference of maximum and minimum values of $r = 25$.

21. (2) : $\because\ 27^{10} + 7^{51} = 27^{10} - 7^{10} + 7^{10} + 7^{51}$
$= 7^{10} + 7^{51}$
[$\because$ The difference of any integral powers with equal exponents is divisible by the difference of their basis.
$\therefore\ 27^{10} - 7^{10}$ is divisible by 20, so it is divisible by 10 also, for remainder 0]
$= (7^2)^5 + 7(7^2)^{25} = (50-1)^5 + 7(50-1)^{25}$
$= (10\alpha - 1) + 7(10\alpha_1 - 1)$
(These are form, not the actual values)
$= 10(\alpha + 7\alpha_1) - 8$
$= 10(\alpha + 7\alpha_1 - 1) + 2 = 10\lambda + 2$, where $\lambda \in I^+$
$\Rightarrow$ Remainder = 2

22. (720) : Number of groups whose sum of digits is divisible by 3 is 0, 1, 2, 4, 5 and 1, 2, 4, 5, 6
$\therefore$ Number of 5 digit numbers divisible by 3 with group 0, 1, 2, 4, 5 = 5! – 4! = 96
Number of 5 digit numbers divisible by 3 using the group 1, 2, 4, 5, 6 = 5! = 120
$\therefore$ Required number of 5 digit numbers which are divisible by 3 $= 96 + 120 = 216 = 6^3 = n^3$ (given)
$\therefore\ n = 6$
Thus, $n! = 6! = 720$

23. (60) : Consider two particular boys as one unit, we have 5 boys



No girl between them so, B_3, B_4 considered as a single boy

$\therefore$ Number of ways in which 6 boys can be seated at a round table when two particular boys are always together = 4! 2!
As, there is no seat between two particular boys, so, we are left with 5 places for girls and these 5 girls can be arranged themselves in 5! ways = 120
$\therefore$ Required ways = Number of arrangements of boys × number of arrangements of girls
$= 4!\,2! \times 5! = 24 \times 2 \times 120 = 10 \times (24)^2 = 10\,(n)^2$
$\therefore\ n = 24 \Rightarrow$ sum of divisors of $n = 60$.

24. (58) : Total number of words formed from the letters of 'SCHOOL' $= \frac{6!}{2!} = 360$
Alphabetic order of letters is C, H, L, O and S.
$\therefore$ Number of words start with C, H, L each $= \frac{5!}{2!} = 60$
Number of words start with O = 5! = 120
$\therefore$ Total words before the letter S = 3 × 60 + 120 = 300
Now, first word start with S *i.e.*, 301th word is SCHLOO and 302th word is SCHOLO and next word is SCHOOL, whose rank is 303. But we need the rank of the word SCHOOL from last.
$\therefore$ Rank of the word SCHOOL from the last
= 360 – 303 + 1 = 58

25. (272) : There are 11 letters in both the words.
In 'EXAMINATION', $\boxed{A = 2, N = 2, I = 2}$ and 5 others are distinct
In 'MATHEMATICS', $\boxed{A = 2, T = 2, M = 2}$ and 5 others are distinct
Number of selection of letters from each word in each case is 4.
$\therefore$ The number of combination in each case will be same.
$\because$ 3 letters are repeated twice and 5 letters are distinct in each case
$\therefore L_1 = L_2$ = coefficient of x^4 in $(x^0 + x^1 + x^2)^3\,(1+x)^5$
= Coefficient of x^4 in $[1 + x(1+x)]^3\,(1+x)^5$
= Coefficient of x^4 in $[1 + x^3(1+x)^3 + 3x(1+x)\,(1 + x(1+x))]\,(1+x)^5$
= Coefficient of x^4 in $[(1+x)^5 + x^3(1+x)^8 + 3x(1+x)^6 + 3x^2(1+x)^7]$
= Coefficient of x^4 in $[(1 + {}^5C_1x^1 + \ldots + {}^5C_4x^4 + \ldots) + x^3(1 + {}^8C_1x^1 + \ldots) + 3x(1 + \ldots + {}^6C_3x^3 + \ldots) + 3x^2(1 + \ldots + {}^7C_2x^2 + \ldots)]$
$= {}^5C_4 + {}^8C_1 + 3 \times {}^6C_3 + 3 \times {}^7C_2$
$= 5 + 8 + 3 \times 20 + 3 \times 21 = 136$
$\therefore\ L_1 + L_2 = 2 \times 136 = 272$

◆◆

beat the TIME TRAP

Duration : 30 minutes

SECTION-I

Single Option Correct Type

1. The value of $\int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1}dx$ equals

(a) $\ln 3$ (b) $2\ln 3$

(c) $\frac{1}{2}\ln 3$ (d) none of these

2. The area bounded by the curve $x^2 + 2x + y - 3 = 0$, the x-axis and the tangent at the point where it meets the y-axis is

(a) $\frac{7}{12}$ sq. units (b) $\frac{12}{7}$ sq. units

(c) $\frac{7}{6}$ sq. units (d) $\frac{6}{7}$ sq. units.

3. If $f(x)=\int_0^x \frac{dt}{(f(t))^2}$ and $\int_0^2 \frac{dt}{(f(t))^2}=\sqrt[3]{6}$, then $f(9)=$

(a) 2 (b) 0

(c) 3 (d) none of these

4. The order and degree, respectively of the differential equation whose solution is $ax^2 + by^2 = 1$, a and b being arbitrary constants are

(a) 1, 1 (b) 2, 1 (c) 2, 2 (d) 1, 2

5. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b}=24$, then $|\vec{u}|^2$ is equal to

(a) 336 (b) 315

(c) 256 (d) 84

6. Let $\frac{d}{dx}(f(x))=\frac{e^{\sin x}}{x}, x>0$. If $\int_1^4 \frac{2e^{\sin x^2}}{x}dx$ $=f(k)-f(1)$, then one of the possible value of k is

(a) -4 (b) 0 (c) 2 (d) 16

7. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots.+\left[\frac{15}{x}\right]\right)$$

(a) does not exist in R (b) is equal to 0

(c) is equal to 15 (d) is equal to 120

8. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is

(a) $\frac{9}{2}$ (b) 6 (c) $\frac{7}{2}$ (d) 4

9. Let $g(x) = \cos x^2$, $f(x)=\sqrt{x}$ and α, β ($\alpha < \beta$) be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$ is

(a) $\frac{1}{2}(\sqrt{2}-1)$ (b) $\frac{1}{2}(\sqrt{3}-1)$

(c) $\frac{1}{2}(\sqrt{3}+1)$ (d) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

10. If sum of all the solutions of the equation

$$8\cos x\cdot\left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$$

in $[0, \pi]$ is $k\pi$, then k is equal to

(a) $\frac{20}{9}$ (b) $\frac{2}{3}$ (c) $\frac{13}{9}$ (d) $\frac{8}{9}$

SECTION-II

Numerical Answer Type

11. If $\frac{x^2+1}{x}=1$, then $\left|x^{200}+\frac{1}{x^{200}}\right|$ equals ________ .

12. The number of real value(s) of x for which $\left(\frac{a}{b}\right)^x = x-2x^2-4$, where a, $b > 0$ is ________ .

CONCEPT MAP

STRAIGHT LINES

Class XI

The general equation of a straight line is $ax + by + c = 0$, where a, b, c are real numbers such that a and b are not zero simultaneously. If the line is inclined at an angle θ with positive x-axis, then slope of the line $= m = \tan\theta = -a/b$.

Point-Slope Form

$\frac{(y-y_1)}{(x-x_1)} = m$, where m is the slope and (x_1, y_1) be any given point on the line.

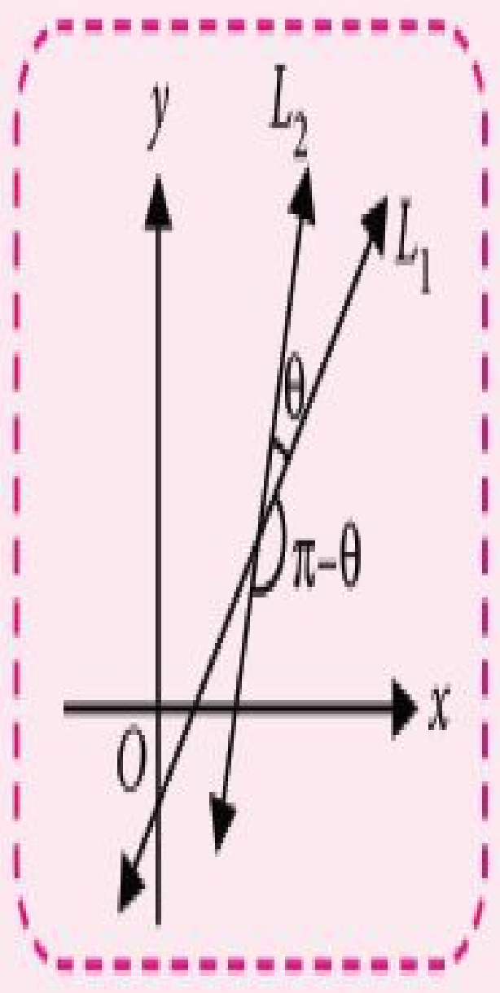
Slope Intercept Form

$y = mx + c$, where m is the slope and c is an intercept on y-axis.

Two Point Form

$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$, where (x_1, y_1) and (x_2, y_2) are any two given points on the line.

Angle Between Two Lines

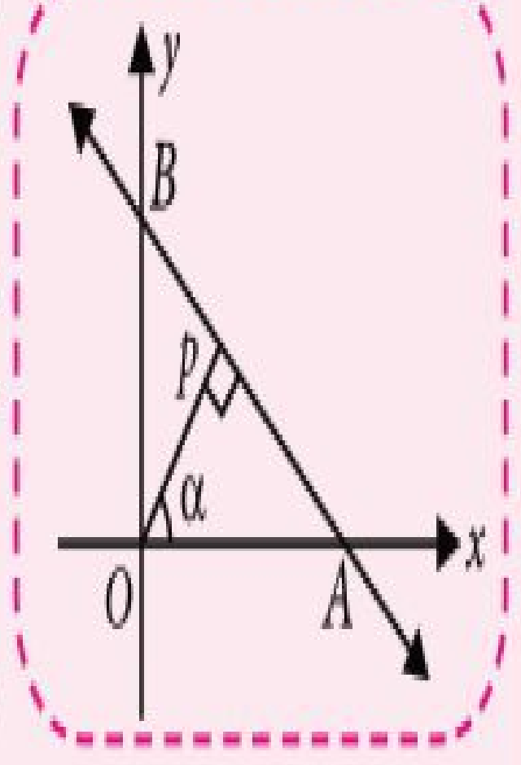


For $L_1 : a_1x + b_1y + c_1 = 0$ and $L_2 : a_2x + b_2y + c_2 = 0$, if acute angle between two lines L_1 and L_2 be θ, then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$, $1 + m_1m_2 \neq 0$

where m_1 = slope of L_1, m_2 = slope of L_2.

- L_1 and L_2 are perpendicular iff $m_1m_2 = -1$.
- L_1 and L_2 are parallel iff $m_1 = m_2$.

Perpendicular (or Normal) Form



$x \cos\alpha + y \sin\alpha = p$, where α is the angle made by perpendicular drawn from $O(0, 0)$ to the line AB with the positive direction of x-axis in anticlockwise sense and P be the length of this perpendicular.

Image of a Point and Foot of Perpendicular

- The image of (x_1, y_1) in the line $ax + by + c = 0$ is given by (x_2, y_2), if $\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$.
- If (x_2, y_2) be the foot of the perpendicular from (x_1, y_1) on the line $ax + by + c = 0$, then $\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$.

Intercept Form

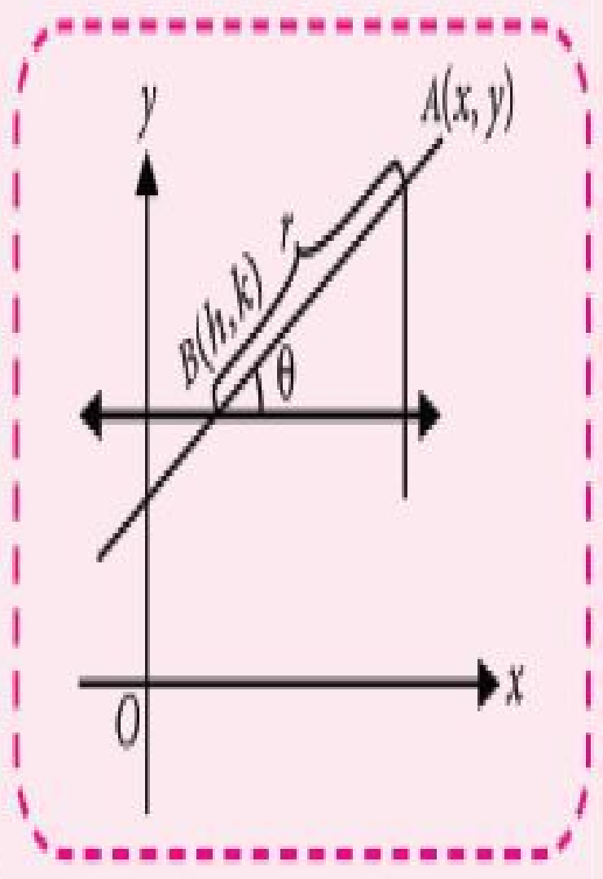
$\frac{x}{a} + \frac{y}{b} = 1$, where a and b represent intercepts on x and y axis respectively.

Parallel Lines

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel iff $a_1/a_2 = b_1/b_2$.

- Distance between two parallel lines *i.e.*, $ax + by + c = 0$ and $ax + by + d = 0$ is $\frac{|c-d|}{\sqrt{a^2 + b^2}}$.

Parametric Form



$\frac{x-h}{\cos\theta} = \frac{y-k}{\sin\theta} = r$, where r is the distance between A and B and $x = h + r\cos\theta$, $y = k + r\sin\theta$.

Perpendicular Lines

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular iff $a_1a_2 + b_1b_2 = 0$.

- Distance of a point (x_1, y_1) from line $ax + by + c = 0$ is given by $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.

Angle Bisector

Let $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ be two lines, then equation of angle bisectors is given by $\left|\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}}\right| = \pm\left|\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}\right|$

CONCEPT MAP

RELATIONS AND FUNCTIONS

Class XII

Relations : R is a relation from A to B if $R \subseteq A \times B$, where A and B are two non-empty sets.

Functions : A function f from a set A to a set B is a rule which associates each element of A to one and only one element of B.

Types of Relations

Empty

A relation R on a set A is called an empty relation, if no element of A is related to any element of A *i.e.*, $R = \phi \subset A \times A$.

Universal

A relation R on a set A is called universal relation, if each element of A is related to every element of A *i.e.*, $R = A \times A$.

Reflexive

A relation $R : A \to A$ is reflexive if xRx $\forall x \in A$.

Symmetric

A relation $R : A \to A$ is symmetric if $xRy \Rightarrow yRx\ \forall x, y \in A$.

Transitive

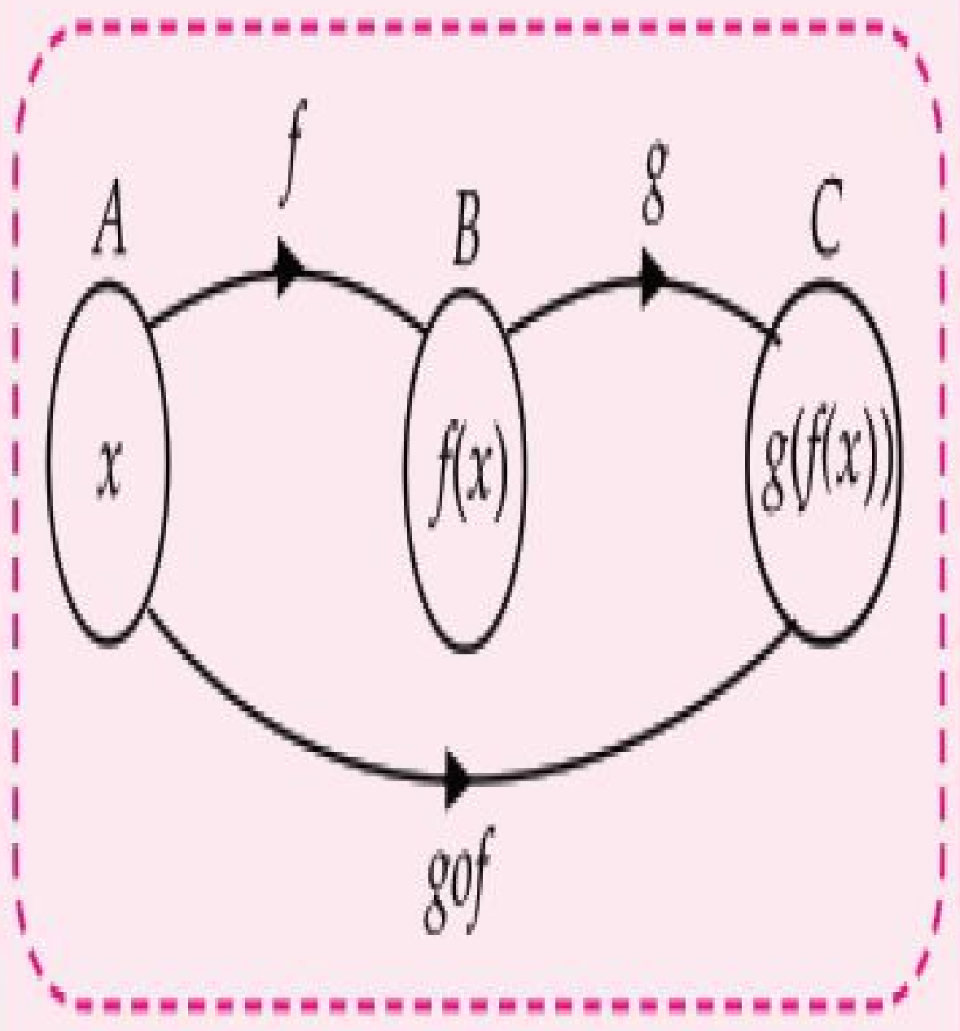
A relation $R : A \to A$ is transitive if xRy and $yRz \Rightarrow xRz\ \forall x, y, z \in A$.

Equivalence

A relation R is said to be an equivalence relation if R is reflexive, symmetric and transitive.

Composition of Functions

Let $f : A \to B$ and $g : B \to C$ be two functions. Then, the function $gof : A \to C$ defined by $(gof)(x) = g(f(x))\ \forall x \in A$ is called the composition of the two functions f and g.



Thus the image of every $x \in A$ under the function gof is the g-image of the f-image of x.

Properties :

- If f and g are injective, then gof is also injective.
- If f and g are surjective, then gof is also surjective.
- If gof is 1 – 1, then f is 1 – 1.
- If $f : X \to Y$, $g : Y \to Z$ and $h : Z \to T$ are functions, then $ho(gof) = (hog)of$ *i.e.*, composition of functions is associative.

Invertible Functions

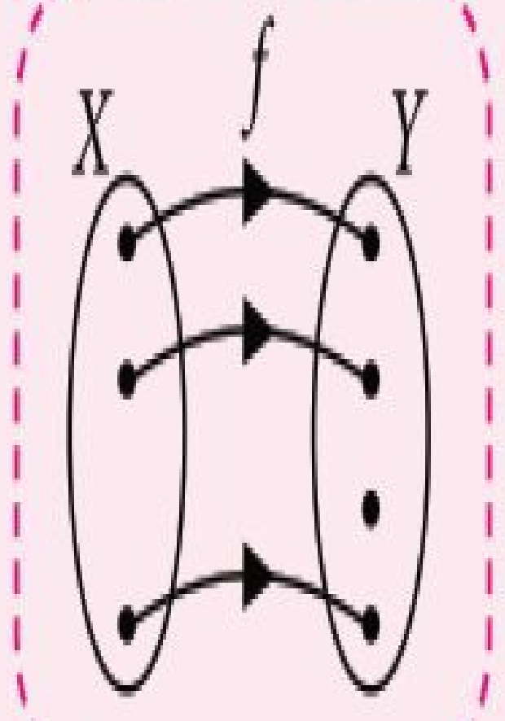
A function $f : X \to Y$ is invertible, if $\exists$ a function $g : Y \to X$ such that $gof = I_x$ and $fog = I_y$, then g is called the inverse of f.

- f is invertible, iff it is both one-one and onto.
- If $f : X \to Y$ and $g : Y \to Z$ be two invertible functions, then gof is invertible and $(gof)^{-1} = f^{-1}og^{-1}$.

Types of Functions

One-one (Injective)

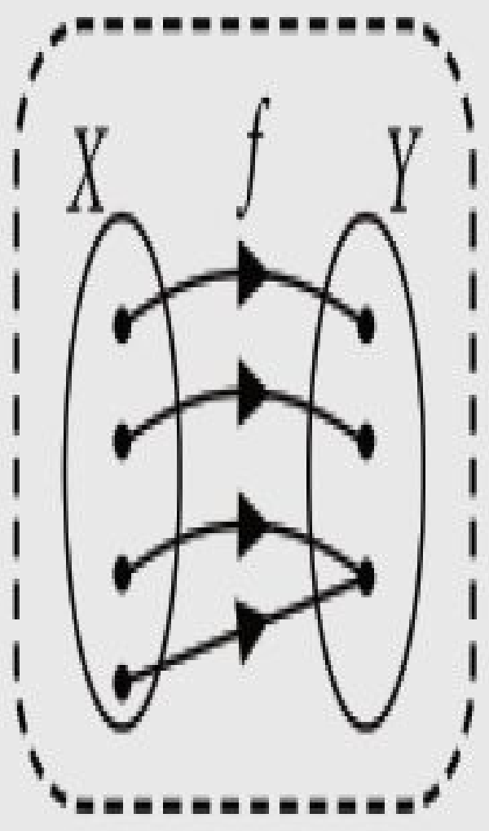
A function $f : X \to Y$ is one-one, if different elements of X have different images in Y under f.



- If two or more than two elements of set X have the same image in Y, then f is called many-one function.

Onto (Surjective)

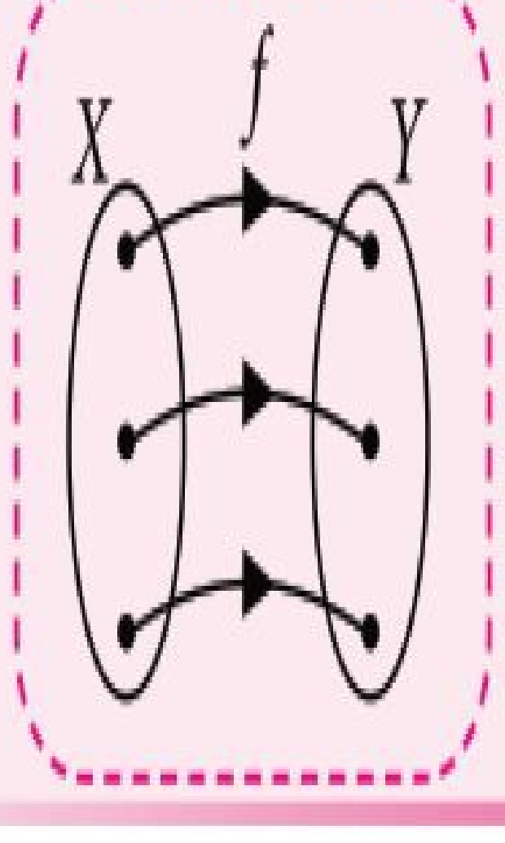
A function $f : X \to Y$ is onto, if every element of Y is the image of some element of X under f.



- If there exists an element in Y having no pre-image in X, then f is called into function.

Bijective

A function $f : X \to Y$ is bijective, if it is both one-one and onto.



13. The number of ways in which 4 different letters from 2*A*'s, 2*T*'s, 2*M*'s, 1*H*, 1*C*, 1*S*, 1*E*, 1*I* can be arranged is ________ .

14. If family of curves satisfy $(9-x^2)\left(\frac{dy}{dx}\right)^2=9-y^2$ and infinite members of the family touch the line $ax + by = 1$, form m ordered pairs of a and b, then m is equal to ________ .

15. If the line through the points $A(4, 1, 2)$, $B(5, \lambda, 0)$ is parallel to the line joining the points $C(2, 1, 1)$ and $D(3, 3, -1)$, then the value of λ equals ________ .

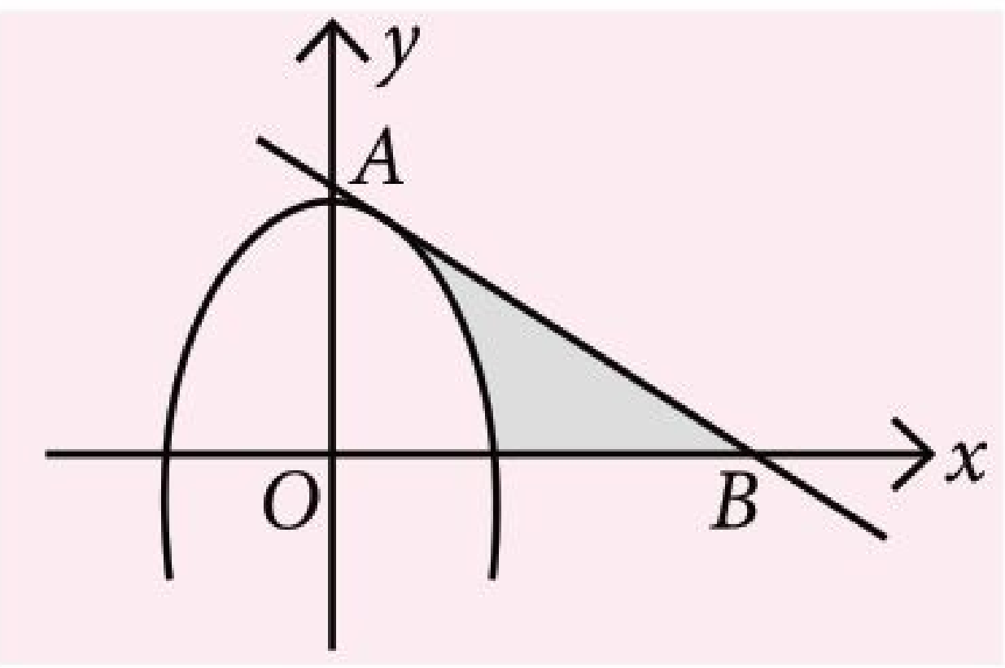
SOLUTIONS

1. (d) : $\int_{-1}^{1}\frac{x^3+|x|+1}{x^2+2|x|+1}dx$

$$=\int_{-1}^{1}\frac{x^3dx}{x^2+2|x|+1}+\int_{-1}^{1}\frac{|x|+1}{x^2+2|x|+1}dx$$

$$=0+2\int_{0}^{1}\frac{|x|+1}{x^2+2|x|+1}=2\int_{0}^{1}\frac{dx}{1+x}=2\ln 2$$

2. (a) : The equation of the curve is $y - 4 = -(x + 1)^2$. The curve meets y-axis at$(0, 3)$ and x-axis at $(-3, 0)$and $(1, 0)$. The tangent at $(0, 3)$ is $y - 3 = -2x$



$$\therefore\ \text{Required area}=\text{Area of }\Delta OAB-\int_{0}^{1}\{4-(x+1)^2\}dx$$

$$=\frac{9}{4}-\left\{4x-\frac{(x+1)^3}{3}\right\}_0^1=\frac{9}{4}-\left\{\left(4-\frac{8}{3}\right)+\frac{1}{3}\right\}=\frac{7}{12}$$

3. (c) : Here, $f(x)=\int_{0}^{x}\frac{dt}{(f(t))^2}\Rightarrow f'(x)=\frac{1}{(f(x))^2}$

$$\Rightarrow\ f'(x)(f(x))^2=1\ \Rightarrow\ \frac{(f(x))^3}{3}=x+c$$

Since $f(2)=\sqrt[3]{6}\ \Rightarrow\ \frac{6}{3}=2+c\ \Rightarrow\ c=0$

$$\therefore\ f(x)=\sqrt[3]{3x}\ \Rightarrow\ f(9)=3$$

4. (b)

5. (a) : Let $\vec{u}=x\hat{i}+y\hat{j}+z\hat{k}$

$\vec{u}\cdot\vec{a}=0$ gives $2x + 3y - z = 0$ ...(i)

$\vec{u}\cdot\vec{b}=24$ gives $y + z = 24$ ...(ii)

Also $\vec{u}$ is coplanar with $\vec{a}$ and $\vec{b}$, so $\left[\vec{u}\ \vec{a}\ \vec{b}\right]=0$,

which yields $\begin{vmatrix}x & y & z\\ 2 & 3 & -1\\ 0 & 1 & 1\end{vmatrix}=0$

$\Rightarrow\ 4x - 2y + 2z = 0$ ...(iii)

(ii) and (iii) gives $2x + 2z = 24 \Rightarrow x + z = 12$

From (i), we get $z = 16$ and thus $x = -4$ and $y = 8$.

Hence, $\vec{u}=-4\hat{i}+8\hat{j}+16\hat{k}$

$$|\vec{u}|=4\sqrt{1^2+2^2+4^2}=4\sqrt{21}\ \ \therefore\ |\vec{u}|^2=336$$

Alternative solution

As $\vec{u}$ is coplanar with $\vec{a}$ and $\vec{b}$, we have $\vec{u}=\lambda\vec{a}+\mu\vec{b}$ for scalars λ and μ.

$\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$ gives

$\vec{u}=2\lambda\hat{i}+(3\lambda+\mu)\hat{j}+(\mu-\lambda)\hat{k}$

$\vec{u}\cdot\vec{b}=24$ gives $\lambda + \mu = 12$; $\vec{u}\cdot\vec{a}=0$ gives $7\lambda + \mu = 0$

Solving the above, we get $\lambda = -2$, $\mu = 14$

Hence, $\vec{u}=-4\hat{i}+8\hat{j}+16\hat{k}$

$\therefore\ |\vec{u}|^2 = 336$

6. (d) : Given, $\frac{d}{dx}(f(x))=\frac{e^{\sin x}}{x},\ x>0$

Now, $I=\int_{1}^{4}\frac{2e^{\sin x^2}}{x}dx$

$$=\int_{1}^{4}\frac{2xe^{\sin x^2}}{x^2}dx\quad[\text{Put } t=x^2\ \Rightarrow\ dt=2xdx]$$

$$=\int_{1}^{16}\frac{e^{\sin t}dt}{t}=[f(t)]_1^{16}=f(16)-f(1)$$

$\therefore\ k = 16$

7. (d) : Observe that $t - 1 < [t] \le t$

Applying this to numbers $\frac{1}{x},\frac{2}{x},.......,\frac{15}{x}$ and summing them, we have

$$\sum_{k=1}^{15}\frac{k}{x}-15<\sum_{k=1}^{15}\left[\frac{k}{x}\right]\le\sum_{k=1}^{15}\frac{k}{x}$$

Multiplying throughout by x, we have

$$\sum_{k=1}^{15}k-15x<x\sum_{k=1}^{15}\left[\frac{k}{x}\right]\le\sum_{k=1}^{15}k$$

Putting the limit $x\to 0^+$, we have

$120 < L \le 120$

As the limit from both sides approaches to 120, we have by sandwich principle, the required limit = 120.

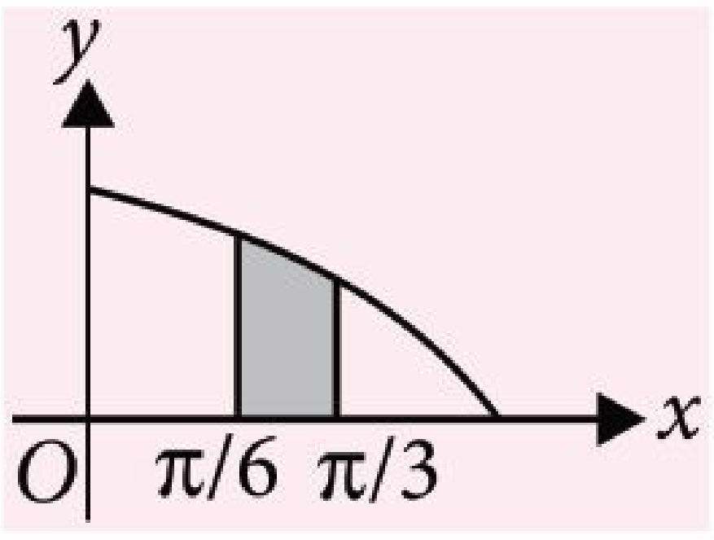
8. (a)

9. (b) : $y = (gof)(x) = \cos\sqrt{x^2} = \cos|x| = \cos x$ [$\because \cos(-x) = \cos x$]

Consider $18x^2 - 9\pi x + \pi^2 = 0$

$\Rightarrow\ (3x - \pi)(6x - \pi) = 0$

$\Rightarrow\ x = \frac{\pi}{3}, \frac{\pi}{6}$



$\therefore$ Required area

$$= \int_{\pi/6}^{\pi/3} \cos x\, dx = [\sin x]_{\pi/6}^{\pi/3} = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{1}{2}(\sqrt{3} - 1)$$

10. (c) : We have,

$8\cos x\ \{\cos(\pi/6 + x)\cos(\pi/6 - x) - 1/2\} = 1$

$$\Rightarrow\ 8\cos x\left\{\cos^2\frac{\pi}{6} - \sin^2 x - 1/2\right\} = 1$$

$[\because\ \cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B]$

$$\Rightarrow\ 8\cos x\left(\frac{3}{4} - 1 + \cos^2 x - \frac{1}{2}\right) = 1$$

$$\Rightarrow\ 8\cos x(\cos^2 x - 3/4) = 1$$

$$\Rightarrow\ 8\cos x\frac{(4\cos^2 x - 3)}{4} = 1\ \Rightarrow 4\cos^3 x - 3\cos x = \frac{1}{2}$$

$$\Rightarrow\ \cos 3x = \frac{1}{2}$$

As $x \in [0, \pi]$

$\Rightarrow\ 3x \in [0, 3\pi]$ which gives

$3x = \pi/3, 5\pi/3, 7\pi/3$

$\therefore\ x = \pi/9, 5\pi/9, 7\pi/9$

which gives sum of all values of $x = \frac{13\pi}{9}$

$\therefore\ k = \frac{13}{9}$

11. (1) : $x + 1/x = 1$

$\Rightarrow\ x^2 - x + 1 = 0$

$\Rightarrow\ (x + \omega)(x + \omega^2) = 0 \qquad \left(\text{where } \omega = \frac{-1 + i\sqrt{3}}{2}\right)$

$\Rightarrow\ x = -\omega, -\omega^2$

When $x = -\omega$, then $x^{200} + \frac{1}{x^{200}} = \omega^{200} + \frac{1}{\omega^{200}}$

$$= (\omega^3)^{66}\cdot\omega^2 + \frac{1}{(\omega^3)^{66}\cdot\omega^2} = \omega^2 + \omega = -1$$

When $x = -\omega^2$, then $x^{200} + \frac{1}{x^{200}} = (-\omega^2)^{200} + \left(\frac{1}{-\omega^2}\right)^{200} = \omega^{400} + \frac{1}{\omega^{400}} = \omega + \omega^2 = -1$

$$\therefore\ \left|x^{200} + \frac{1}{x^{200}}\right| = 1$$

12. (0) : L.H.S. $= \left(\frac{a}{b}\right)^x$, which is $> 0\ [x \in R]$

R.H.S. $= -2x^2 + x - 4$

$$= -2\left(x^2 - \frac{x}{2}\right) - 4 = -2\left[\left(x - \frac{1}{4}\right)^2 - \frac{1}{16}\right] - 4$$

$$= -2\left[\left(x - \frac{1}{4}\right)^2\right] + \frac{1}{8} - 4 = -2\left[\left(x - \frac{1}{4}\right)^2\right] - \frac{31}{8} < 0\ \forall\, x \in R$$

$\therefore$ The given equation has no solution.

13. (2454) : Given letters are the letters of the word Mathematics (MMAATTHEICS)

Number of arrangements = Coefficient of x^4 in

$$4!\left(1 + \frac{x}{1!} + \frac{x^2}{2!}\right)^3 \times \left(1 + \frac{x}{1!}\right)^5$$

[As 3 set of double numbers *i.e.*, MM, AA, TT and 5 letters are distinct.]

= Coefficient of x^4 in

$$4!\left[(1+x)^3 + \frac{x^6}{(2!)^3} + \frac{3}{2}(1+x)^2 x^2 + \frac{3}{4}x^4(1+x)\right](1+x)^5$$

= Coefficient of x^4 in

$$4!\left[(1+x)^8 + \frac{x^6}{8}(1+x)^5 + \frac{3}{2}x^2(1+x)^7 + \frac{3}{4}x^4(1+x)^6\right]$$

$$= 4!\left[{}^8C_4 + \frac{1}{8}\times 0 + \frac{3}{2}\times{}^7C_2 + \frac{3}{4}\times{}^6C_0\right]$$

$$= 4!\left[\frac{8\cdot 7\cdot 6\cdot 5}{4\cdot 3\cdot 2\cdot 1} + \frac{3}{2}\cdot\frac{7}{2}\cdot\frac{6}{1} + \frac{3}{4}\right] = 24\left[70 + \frac{63}{2} + \frac{3}{4}\right]$$

$$= 24\left[\frac{280 + 126 + 3}{4}\right] = 6\ [409] = 2454$$

14. (4) : $(9 - x^2)\left(\frac{dy}{dx}\right)^2 = 9 - y^2$

$$\Rightarrow\ \frac{dy}{\sqrt{9 - y^2}} = \pm\frac{dx}{\sqrt{9 - x^2}}$$

$$\Rightarrow\ \sin^{-1}\frac{y}{3} = \pm\sin^{-1}\frac{x}{3} + c\ \Rightarrow\ \frac{y}{3} = \sin\left(\pm\sin^{-1}\frac{x}{3} + c\right)$$

$$\Rightarrow\ \frac{y}{3} = \sin c\cos\left(\pm\sin^{-1}\frac{x}{3}\right) \pm \frac{x}{3}\cos c$$

$$\Rightarrow\ \frac{y}{3} = \sin c\cos\left(\cos^{-1}\sqrt{1 - \frac{x^2}{9}}\right) \pm \frac{x}{3}\cos c$$

$$\Rightarrow\ y = \sin c\sqrt{9 - x^2} \pm x\cos c$$

$\Rightarrow\ y = k(9 - x^2)^{1/2} \pm hx, \qquad$ (where $h^2 + k^2 = 1$)

which touches $x = 3, x = -3, y = 3, y = -3$

15. (3) : d.r.'s of AB are $< 1, \lambda - 1, -2 >$

d.r.'s of CD are $< 1, 2, -2 >$

As $AB \parallel CD\ \therefore\ 1 = \frac{\lambda - 1}{2} = \frac{-2}{-2}\quad \Rightarrow\quad \lambda = 3$

◆ ◆

YOU ASK
WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. For $r = 0, 1, \ldots\ldots, 10$, let A_r, B_r and C_r denote respectively, the coefficients of x^r in the expansion of $(1 + x)^{10}$, $(1 + x)^{20}$, $(1 + x)^{30}$. Then find the value of $\sum_{r=1}^{10} A_r (B_{10}B_r - C_{10} A_r)$. *(Satyadev Kumar, Bihar)*

Ans. $(1 + x)^{10} = A_0 + A_1x + A_2x^2 + \ldots + A_{10}x^{10}$

$(x + 1)^{20} = B_0x^{20} + B_1x^{19} + B_2x^{18} + \ldots + B_{20}$. Considering the coefficient of x^{20} in the product, we get

$A_0B_0 + A_1B_1 + A_2B_2 + \ldots + A_{10} B_{10}$ = coefficient of x^{20} in the expansion of $(1 + x)^{10}(x + 1)^{20} = (1 + x)^{30}$ which is C_{20}

$$\therefore \quad \sum_{r=0}^{10} A_rB_r = C_{20} = C_{10}$$

$$\text{But } \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \ldots + \binom{n}{n}^2 = \binom{2n}{n}$$

$$\therefore \quad A_0^{\,2} + A_1^{\,2} + A_2^{\,2} + \ldots + A_{10}^{\,2} = \binom{20}{10} = B_{10}$$

$$\Rightarrow \quad \sum_{r=0}^{10} A_r^{\,2} = B_{10}.$$

$$\text{Now, } \sum_{r=1}^{10} A_r (B_{10} B_r - C_{10} A_r)$$

$$= B_{10} \sum_{r=1}^{10} A_r B_r - C_{10} \sum_{r=1}^{10} A_r^{\,2}$$

$$= B_{10} (C_{10} - 1) - C_{10} (B_{10} - 1) = C_{10} - B_{10}.$$

2. Solve for x :

$$\frac{6}{5}a^{\log_a x \log_{10} a \log_a 5} - 3^{\log_{10}(x/10)} = 9^{\log_{100} x + \log_4 2}.$$

(Prachi Sharma, Delhi)

Ans. We have,

$$a^{\log_a x \log_{10} a \log_a 5} = a^{\log_a 5 \log_{10} x} = [a^{\log_a 5}]^{\log_{10} x} = 5^{\log_{10} x}$$

Also, $\log_{10}(x/10) = \log_{10} x - 1$

and $\log_{100} x + \log_4 2 = \dfrac{\log_{10} x}{\log_{10} 100} + \dfrac{\log_{10} 2}{\log_{10} 4} = \dfrac{\log_{10} x}{2} + \dfrac{1}{2}$

The given equation thus reduces to

$$\frac{6}{5}5^{\log_{10} x} - 3^{\log_{10} x - 1} = 9^{(\log_{10} x + 1)/2}$$

$$\Rightarrow \quad \frac{6}{5}5^{\log_{10} x} - 3^{\log_{10} x} \cdot 3^{-1} = 3^{(\log_{10} x + 1)}$$

$$\Rightarrow \quad \frac{6}{5}5^{\log_{10} x} - \frac{1}{3} \cdot 3^{\log_{10} x} = 3^{\log_{10} x} \cdot 3^1$$

$$\Rightarrow \quad \frac{6}{5}5^{\log_{10} x} = \left(3 + \frac{1}{3}\right)3^{\log_{10} x} \Rightarrow \frac{6}{5}5^{\log_{10} x} = \frac{10}{3}\,3^{\log_{10} x}$$

$$\Rightarrow \quad \frac{9}{25} = \left(\frac{3}{5}\right)^{\log_{10} x} \Rightarrow \left(\frac{3}{5}\right)^2 = \left(\frac{3}{5}\right)^{\log_{10} x}$$

$$\Rightarrow \quad \log_{10} x = 2 \;\Rightarrow\; x = 10^2 = 100$$

3. Evaluate : $\displaystyle\int \frac{1}{x}\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$ *(Animesh, West Bengal)*

Ans. Let $I = \displaystyle\int \left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)^{1/2} \frac{dx}{x}$

Put $x = \cos^2 2\theta \Rightarrow dx = 2 \cos 2\theta\,(-2 \sin 2\theta)\, d\theta$

$$\therefore \quad I = \int \left(\frac{1-\cos 2\theta}{1+\cos 2\theta}\right)^{1/2} \times \frac{-4\cos 2\theta \sin 2\theta\, d\theta}{\cos^2 2\theta}$$

$$= -8\int \frac{\sin^2\theta}{\cos 2\theta} d\theta = 4\int\left(\frac{\cos 2\theta - 1}{\cos 2\theta}\right) d\theta = 4\int (1 - \sec 2\theta)\, d\theta$$

$$= 4\theta - 2 \ln |\sec 2\theta + \tan 2\theta| + C$$

$$= 2\cos^{-1}\sqrt{x} - 2\ln\left|\frac{1+\sqrt{1-x}}{\sqrt{x}}\right| + C$$

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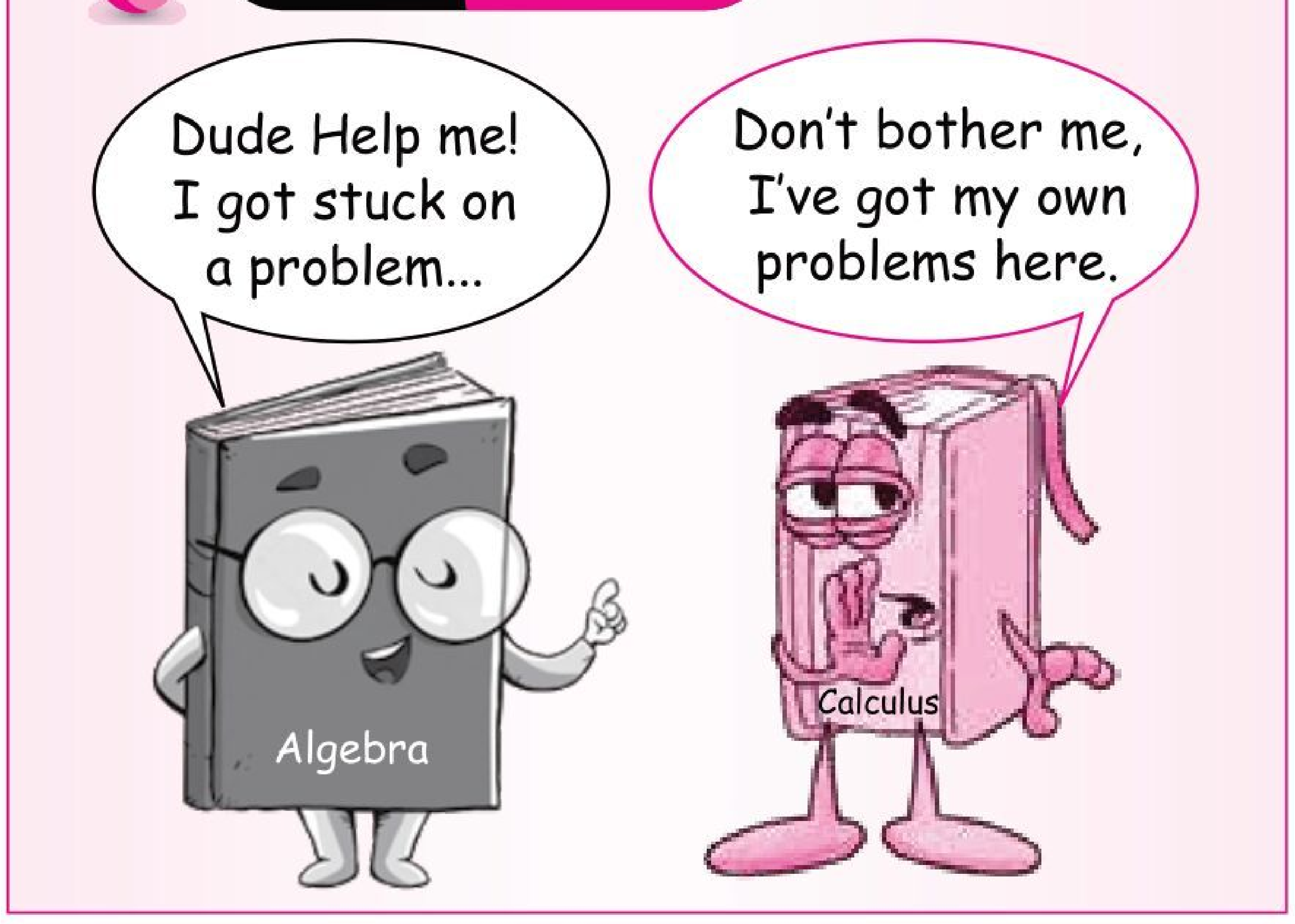
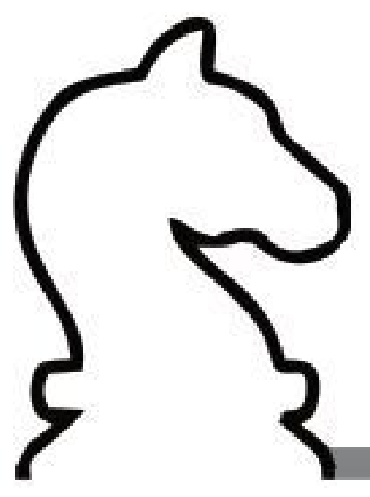
COMIC CAPSULE


Dude Help me! I got stuck on a problem...
Don't bother me, I've got my own problems here.
Algebra
Calculus



Challenging PROBLEMS

ON

JEE

SINGLE CORRECT ANSWER TYPE

1. Let $S = \{a \in N, a \le 100\}$. If the equation $[\tan^2 x] - \tan x - a = 0$ has real roots, then number of elements in S is (where $[\cdot]$ is step function)

(a) 10 (b) 8 (c) 9 (d) 0

2. The equation $2x = (2n+1)\pi(1-\cos x)$, (where n is a positive integer)

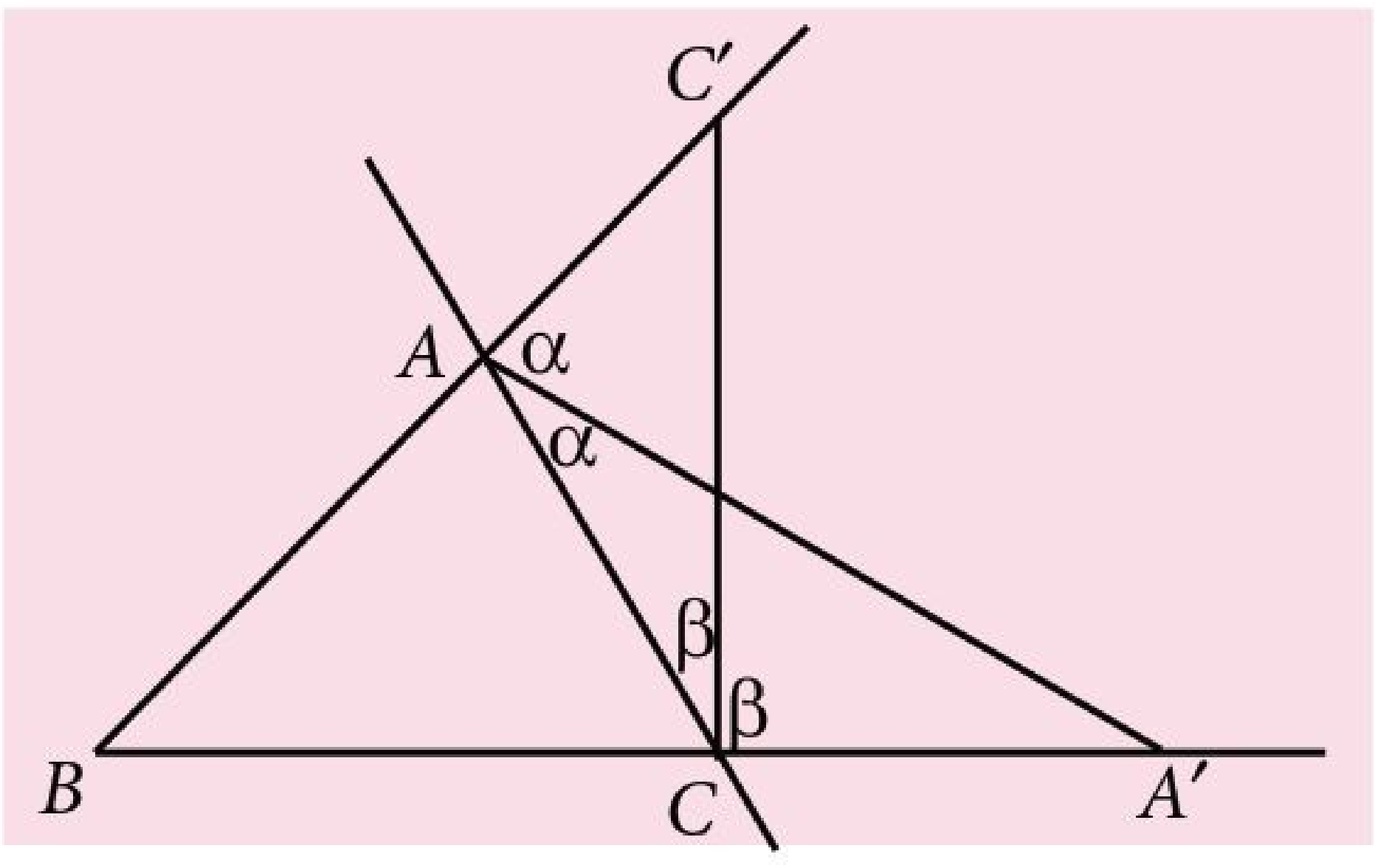
(a) has infinitely many real roots
(b) has exactly one real root
(c) has exactly $2n + 2$ real roots
(d) has exactly $2n + 3$ real roots

3. $|x^2 \sin x + \cos^2 x e^x + \ln^2|x|| < x^2|\sin x| + \cos^2 x e^x + \ln^2|x|$ true for $x \in$

(a) $(-\pi, 0)$ (b) $\left(0, \frac{\pi}{2}\right)$
(c) $\left(\frac{\pi}{2}, \pi\right)$ (d) $(2n\pi, (2n+1)\pi) n \in N$

4. Given in ΔABC, $AB = 1$ cm, $AC = 2$ cm. The lengths of external angular bisectors of angles A and C are equal. *i.e.*, $AA' = CC'$. If $BC \ne 1$, then $BC =$

[In the given figure, $\alpha = 90° - \frac{A}{2}$ and $\beta = 90° - \frac{C}{2}$]



(a) $\frac{1+\sqrt{15}}{2}$ (b) $\frac{1+\sqrt{13}}{2}$
(c) $\frac{1+\sqrt{17}}{2}$ (d) $\frac{1+\sqrt{19}}{2}$

5. In ΔABC, the bisector of the angle A meets the side BC at D and the circumscribed circle at E, then DE equals

(a) $\frac{a^2 \sec\frac{A}{2}}{2(b+c)}$ (b) $\frac{a^2 \sin\frac{A}{2}}{2(b+c)}$
(c) $\frac{a^2 \cos\frac{A}{2}}{2(b+c)}$ (d) $\frac{a^2 \operatorname{cosec}\frac{A}{2}}{2(b+c)}$

6. Let there exist a unique point P inside a ΔABC such that $\angle PAB = \angle PBC = \angle PCA = \alpha$. If $PA = x$, $PB = y$, $PC = z$, Δ = area of ΔABC and a, b, c are the sides opposite to the angles A, B, C respectively, then $\tan\alpha$ is equal to

(a) $\frac{a^2+b^2+c^2}{4\Delta}$ (b) $\frac{a^2+b^2+c^2}{2\Delta}$
(c) $\frac{2\Delta}{a^2+b^2+c^2}$ (d) $\frac{4\Delta}{a^2+b^2+c^2}$

7. If a, b, c are the sides of a triangle, then the minimum value of $\frac{2a}{b+c-a} + \frac{2b}{c+a-b} + \frac{2c}{a+b-c}$ is

(a) 3 (b) 6 (c) 8 (d) 1/8

8. If length of the sides of a triangle ABC are 3, 4 and 5 cm, then distance between its orthocentre and circumcentre is

(a) 2.5 c.m. (b) 2 c.m.
(c) 1.5 c.m. (d) 8 c.m.

9. Three positive real numbers x, y, z satisfy the equations $x^2 + \sqrt{3}xy + y^2 = 25$, $y^2 + z^2 = 9$ and $x^2 + xz + z^2 = 16$, then the value of $xy + 2yz + \sqrt{3}xz$ is

(a) 18 (b) 24 (c) 30 (d) 36

10. In the triangle ABC the medians from B and C are perpendicular. The value of $\cot B + \cot C$ cannot be

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

By : Tapas Kr. Yogi (Prime Axis Classes), Visakhapatnam Mob : 09533632105, 09949428844

11. In a scalene ΔABC, D is a point on the side AB such that $CD^2 = AD \cdot DB$, if $\sin A \cdot \sin B = \sin^2\dfrac{C}{2}$, then CD is

(a) Median through C
(b) Internal bisector of C
(c) Altitude through C
(d) Divides AB in the ratio 1 : 2

12. If in a ΔABC the incentre and circumcentre are reflections of each other in side BC, then the measure of $\angle BAC$ (in degrees) is

(a) 120 (b) 108 (c) 135 (d) 105

13. Let I be the incentre of the triangle ABC, where $\dfrac{\overrightarrow{BC}}{|\overrightarrow{BC}|}+\dfrac{\overrightarrow{BA}}{|\overrightarrow{BA}|}=\dfrac{\overrightarrow{BI}}{k}$, then the diameter of the circumcircle of the triangle is

(a) $k\left(\cos\dfrac{A}{2}+\cos\dfrac{C}{2}\right)$ (b) $k\left(\sin\dfrac{A}{2}+\sin\dfrac{C}{2}\right)$

(c) $k\left(\cot\dfrac{A}{2}+\cot\dfrac{C}{2}\right)$ (d) $k\left(\tan\dfrac{A}{2}+\tan\dfrac{C}{2}\right)$

14. In an isosceles triangle ABC, $AB = AC$. If vertical angle A is 20°, then $a^3 + b^3$ is equal to

(a) $3a^2b$ (b) $3b^2c$ (c) $3c^2a$ (d) abc

15. Consider a given acute angled triangle ABC having O as its circumcentre. Let D be a variable interior point of the side BC. The limiting value of the circumradius of the ΔOCD as point D approaches towards vertex C is equal to

(a) $\dfrac{R}{2\cos A}$ (b) $\dfrac{R}{\cos A}$

(c) $\dfrac{R}{\sin A}$ (d) $\dfrac{R}{2\sin A}$

MULTIPLE ANSWER CORRECT TYPE

16. Let x, y, z be real numbers with $x \geq y \geq z \geq \dfrac{\pi}{12}$ such that $x+y+z=\dfrac{\pi}{2}$ and let $P = \cos x \cdot \sin y \cdot \cos z$, then

(a) Minimum value of P is $\dfrac{1}{8}$

(b) Minimum value of P is $\dfrac{1}{4}$

(c) Maximum value of P is $\dfrac{2+\sqrt{3}}{4}$

(d) Maximum value of P is $\dfrac{2+\sqrt{3}}{8}$

17. For $0<\theta<\dfrac{\pi}{2}$, the solution(s) of

$$\sum_{m=1}^{6}\operatorname{cosec}\left(\theta+\frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta+\frac{m\pi}{4}\right)=4\sqrt{2}$$ is/are

(a) $\pi/4$ (b) $\pi/6$ (c) $\pi/12$ (d) $5\pi/12$

18. Given an isosceles triangle with equal side of length b, base angle $\alpha < \pi/4$. R, r the radii and O, I the centres of the circumcircle and incircle, respectively. Then

(a) $R=\dfrac{1}{2}b\operatorname{cosec}\alpha$ (b) $R=\dfrac{2}{3}b\cos\alpha$

(c) $r=\dfrac{b\sin 2\alpha}{2(1+\cos\alpha)}$ (d) $OI=\left|\dfrac{b\cos(3\alpha/2)}{2\sin\alpha\cos(\alpha/2)}\right|$

19. In a triangle ABC, perpendiculars drawn from vertices A, B, C meet the opposite sides BC, CA, AB at D, E, F respectively, triangle DEF is completed. The perimeter of triangle DEF is greater than or equal to $3\sqrt{3}\,r$, where r is in-radius of triangle ABC. Also $r=\sqrt{3}$ and perimeter of triangle ABC is 18. Then

(a) triangle ABC is right angled
(b) triangle ABC is equilateral
(c) area of triangle ABC is $9\sqrt{3}$
(d) ratio of area of triangle ABC to triangle DEF is 4 : 1

COMPREHENSION TYPE

Paragraph for Q. No. 20 and 21

In triangle ABC, $BC = a$, $CA = b$, $AB = c$. R is the circumradius and r is in-radius and s is the semi perimeter and it is given that $\left(\cot\dfrac{A}{2}\right)^2+\left(2\cot\dfrac{B}{2}\right)^2+\left(3\cot\dfrac{C}{2}\right)^2=\left(\dfrac{6s}{7r}\right)^2$.

20. The ratio of the sides of triangle ABC, $a : b : c$ is

(a) 1 : 1 : 1 (b) 13 : 40 : 45
(c) 1 : 2 : 3 (d) 7 : 15 : 45

21. The greatest angle of the triangle is

(a) $\pi-\cos^{-1}\left(\dfrac{16}{65}\right)$ (b) $\sin^{-1}\left(\dfrac{16}{65}\right)$

(c) $\cos^{-1}\left(\dfrac{63}{65}\right)$ (d) None of these

INTEGER ANSWER TYPE

22. If $[\sin x]+\left[\dfrac{x}{2\pi}\right]+\left[\dfrac{2x}{5\pi}\right]=\dfrac{9x}{10\pi}$, when the number of solutions in the interval (30, 40) is (where [·] is GIF) __________.

23. Let 'k' be sum of all 'x' in the interval $[0, 2\pi]$ such that $3\cot^2 x + 8\cot x + 3 = 0$. Then the value of k/π is ________.

24. The number of solutions of equation $8[x^2 - x] + 4[x] = 13 + 12[\sin x]$ is (where $[\cdot]$ represents greatest integer less than or equal to x) ________.

25. If the circumcentre of triangle ABC lies on its incircle, then $[4(\cos A + \cos B + \cos C)]$ is (where $[\cdot]$ is greatest integer less than or equal to x) ________.

26. Given a parallelogram whose acute angle is 60°. If the squares of length of the diagonals are in ratio 1 : 3, then $\frac{a}{b}$ is equal to (where a, b are the lengths of the sides) _____.

27. ABC is an acute angle triangle, $\angle A = 30°$. H is the orthocentre and M is the midpoint of BC. On the line HM a point T is taken such that $HM = MT$. If BC = 4 cm, then the length of AT is ________

SOLUTIONS

1. (c) : Given equation is true only when $\tan x$ is an integer.

$\tan x = \frac{1 \pm \sqrt{4a+1}}{2}$, where $D = 4a + 1$ is perfect square and $4a + 1 \le 401$

2. (c) : $\sin^2\left(\frac{x}{2}\right) = \frac{x}{(2n+1)\pi}$

The graph of $\sin^2\left(\frac{x}{2}\right)$ will be above the x–axis and meet the x-axis at $0, 2\pi, 4\pi, \ldots$ etc. It will attain maximum values at odd multiples of π *i.e.*, $\pi, 3\pi, \ldots, (2n + 1)\pi$.

The last point after which graph of $y = \frac{x}{(2n+1)\pi}$ will stop cutting will be $(2n + 1)\pi$.

$\therefore$ Total intersection $= 2(n + 1)$

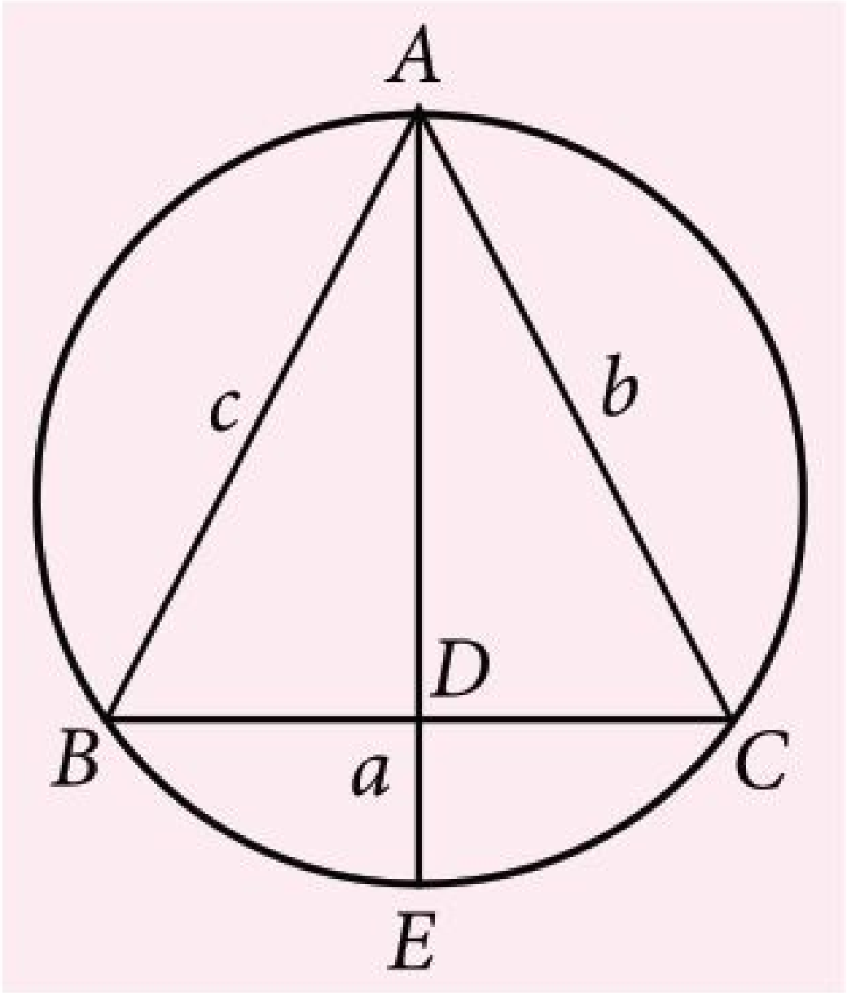
3. (a) : $\because |a + b + c| < |a| + |b| + |c|$

Also, a, b, c does not have same sign.

So, $x^2 \sin x < 0$, if $x \in (-\pi, 0)$

4. (c) : Length of external angular bisector of angle A is $\frac{2bc}{|b-c|}\sin\frac{A}{2}$. Length of external angular bisector of angle C is $\frac{2ab}{|a-b|}\sin\frac{c}{2}$.

5. (a) :



$AD \cdot DE = BD \cdot DC$

$$\Rightarrow DE = \frac{BD \cdot DC}{AD} = \frac{\left(\frac{ac}{b+c}\right)\left(\frac{ab}{b+c}\right)}{\frac{2bc}{b+c} \cdot \cos\frac{A}{2}} = \frac{a^2}{2(b+c)}\sec\frac{A}{2}$$

6. (d) : $\cos\alpha = \frac{a^2 + b^2 + c^2}{2cx + 2ay + 2bz}$ and

$\sin\alpha = \frac{2\Delta}{ay + cx + bz}$

$\therefore \quad \tan\alpha = \frac{4\Delta}{a^2 + b^2 + c^2}$

7. (b) : Given expression can be written as

$$\frac{a}{s-a} + \frac{b}{s-b} + \frac{c}{s-c}$$

$$= -3 + \left(\frac{s}{s-a} + \frac{s}{s-b} + \frac{s}{s-c}\right) \ge -3 + 9 = 6$$

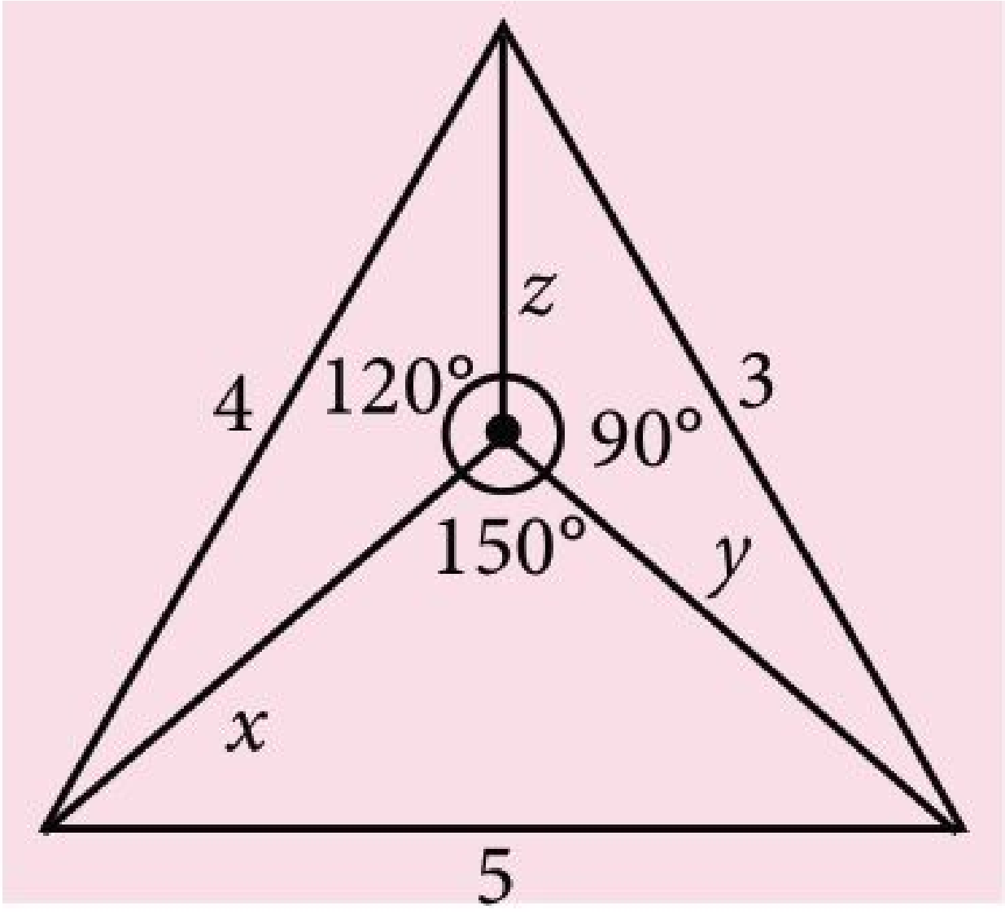
$$\left(\because (x_1 + x_2 + x_3)\left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}\right) \ge 9\right)$$

8. (a) : Since, distance between orthocentre and circumcentre is circumradius.

$\therefore \quad R = \frac{abc}{4\Delta} = \frac{3 \cdot 4 \cdot 5}{4\left(\frac{1}{2} \times 3 \times 4\right)}$ [$\because ABC$ is rt Δ]

$= 2.5$

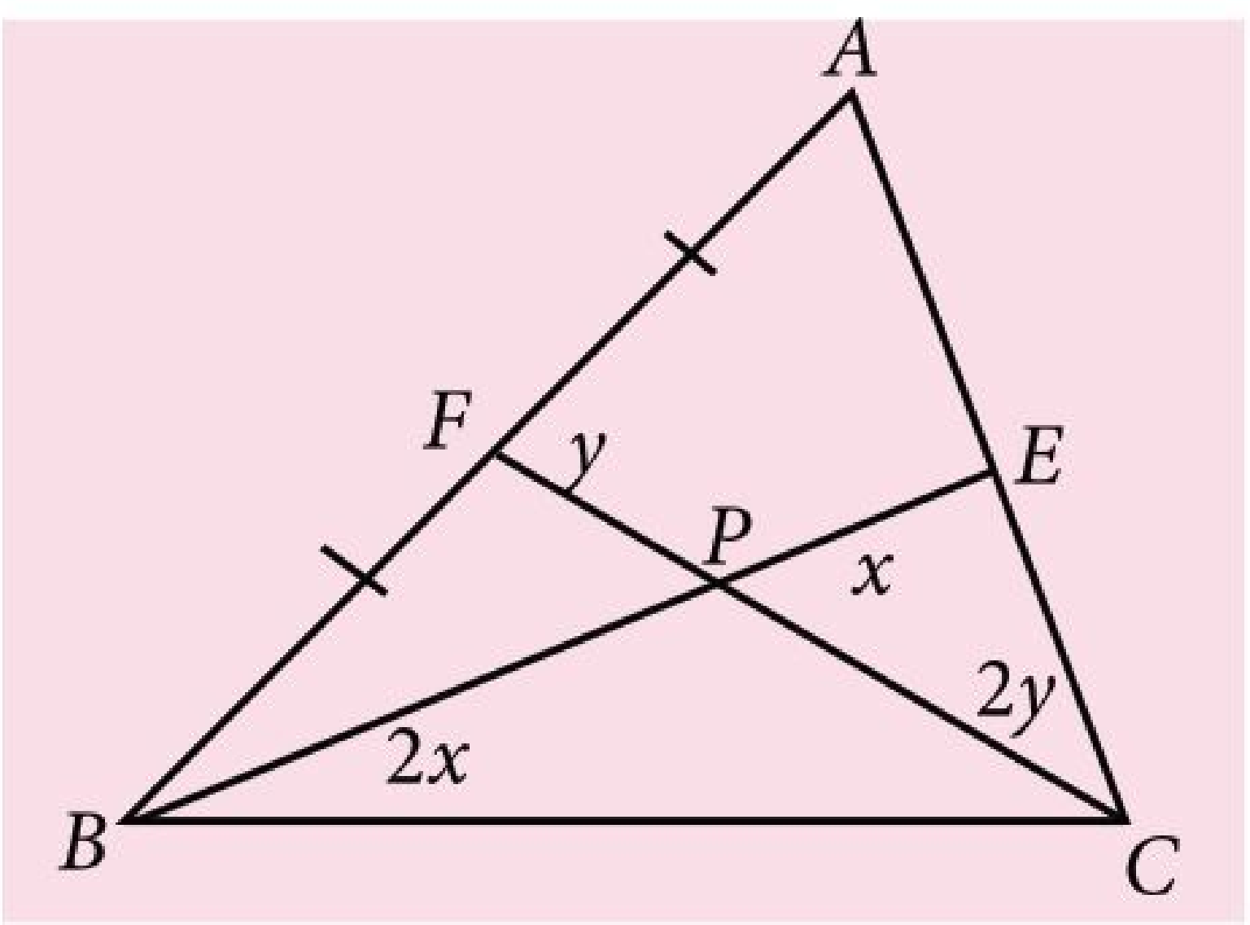
9. (b) :



Area of triangle $= \frac{1}{2} \times 3 \times 4 = \frac{1}{2}xz\frac{\sqrt{3}}{2} + \frac{1}{2}xy \times \frac{1}{2} + \frac{1}{2}yz$

$\Rightarrow \quad 24 = \sqrt{3}xz + xy + 2yz$

10. (b) :



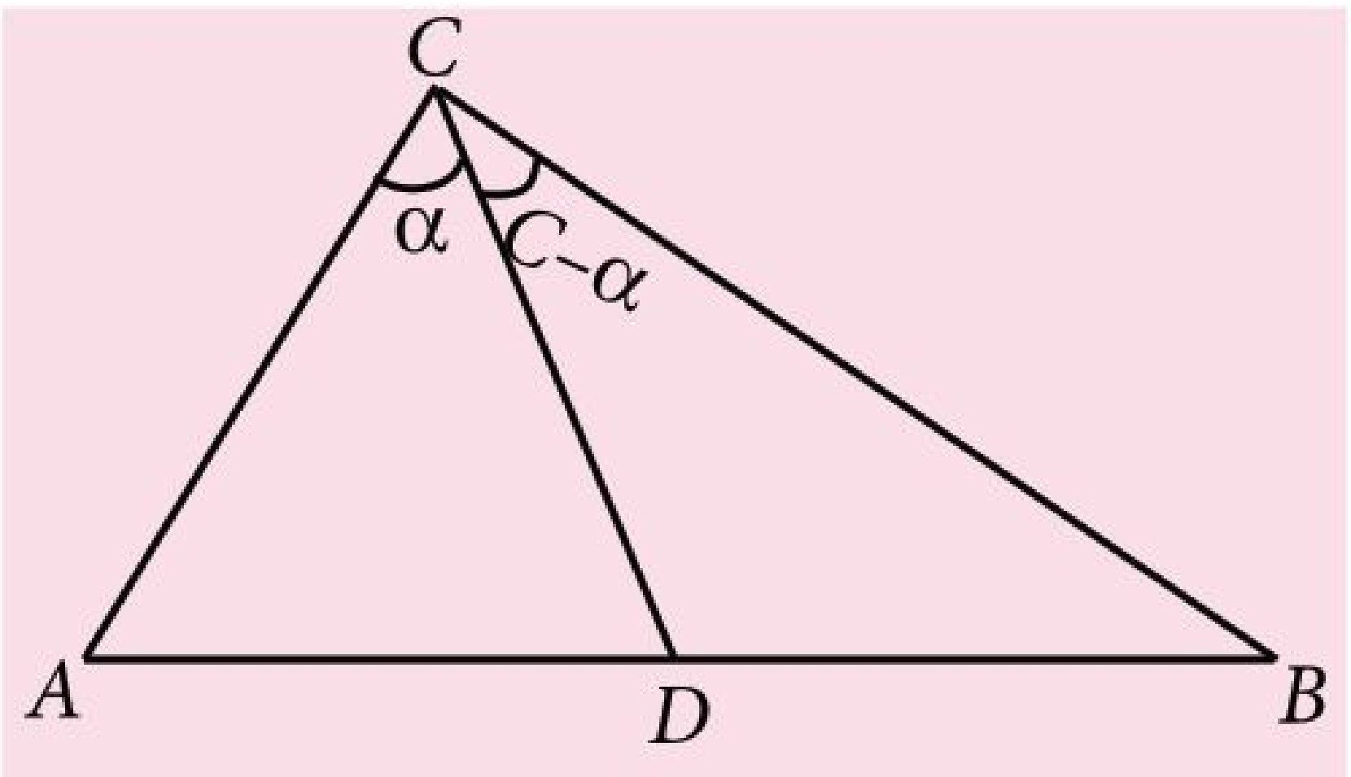
Here, $\tan\angle PBF = \frac{y}{2x}$, $\tan\angle PBC = \frac{y}{x}$

$$\therefore \quad \tan B = \frac{\frac{y}{2x}+\frac{y}{x}}{1-\frac{y^2}{2x^2}} = \frac{3xy}{2x^2-y^2}$$

$$\Rightarrow \quad \cot B = \frac{2x^2-y^2}{3xy}, \ \cot C = \frac{2y^2-x^2}{3xy}$$

$$\therefore \quad \cot B + \cot C = \frac{x^2+y^2}{3xy} \geq \frac{2}{3}$$

11. (b) : Let $\angle ACD = \alpha \Rightarrow \angle DCB = C - \alpha$



Applying the sine rule in ΔACD and ΔDCB respectively, we get

$$\frac{AD}{\sin\alpha} = \frac{CD}{\sin A} \text{ and } \frac{BD}{\sin(C-\alpha)} = \frac{CD}{\sin B}$$

$$\Rightarrow \quad \frac{AD \cdot BD}{\sin\alpha \cdot \sin(C-\alpha)} = \frac{CD^2}{\sin A \cdot \sin B}$$

$$\Rightarrow \quad \frac{1}{2}[\cos(2\alpha - C) - \cos C] = \sin^2\frac{C}{2}$$

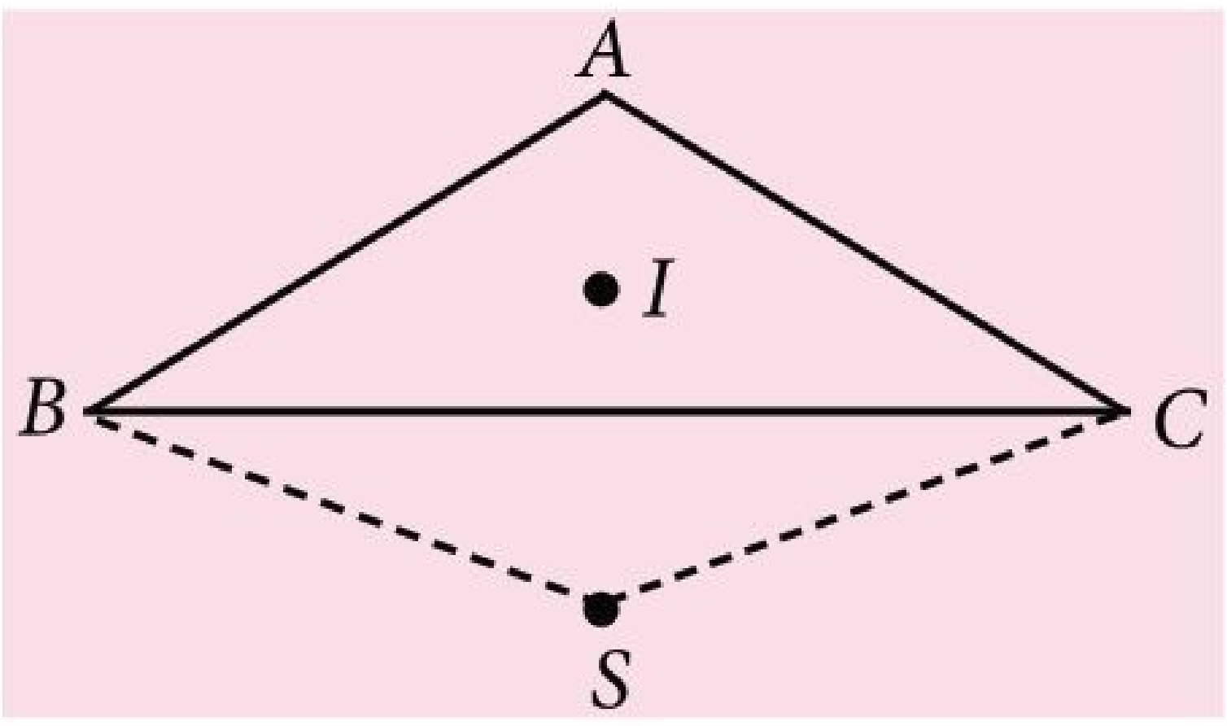
$$\Rightarrow \quad \frac{1}{2}\left[\cos(2\alpha - C) - 1 + 2\sin^2\frac{C}{2}\right] = \sin^2\frac{C}{2}$$

$$\Rightarrow \quad \frac{1}{2}\cos(2\alpha - C) - \frac{1}{2} = 0 \ \Rightarrow \ \cos(2\alpha - C) = 1$$

$$\Rightarrow \quad 2\alpha - C = 0 \Rightarrow \quad \alpha = \frac{C}{2}$$

$\therefore$ CD is the internal angle bisector of angle C.

12. (b) :



Here, I is the incentre and S is the circumcentre.

$$\angle BIC = 90° + \frac{A}{2}$$

and reflex $\angle BSC = 2A \Rightarrow \angle BSC = 360° - 2A$

Hence $90° + \frac{A}{2} = 360° - 2A \ \Rightarrow \angle A = 108°$

13. (c) : Taking modulus both sides, we get

$$2\cos\frac{B}{2} = \frac{1}{k}|\overrightarrow{BI}| = \frac{1}{k}\frac{r}{\sin(B/2)} = \frac{4R\sin(A/2)\sin(C/2)}{k}$$

$$\Rightarrow \quad 2R = \frac{k\sin\left(\frac{A+C}{2}\right)}{\sin(A/2)\sin(C/2)} = k\left(\cot\frac{A}{2} + \cot\frac{C}{2}\right)$$

14. (c) : $\because \angle A = 20°$

$\therefore$ $\angle B = \angle C = 80°$ and $b = c$

Now, $\frac{a}{\sin 20°} = \frac{b}{\sin 80°} = \frac{c}{\sin 80°}$

$$\Rightarrow \quad \frac{a}{\sin 20°} = \frac{b}{\cos 10°} \quad \Rightarrow \quad a = 2b\sin 10°$$

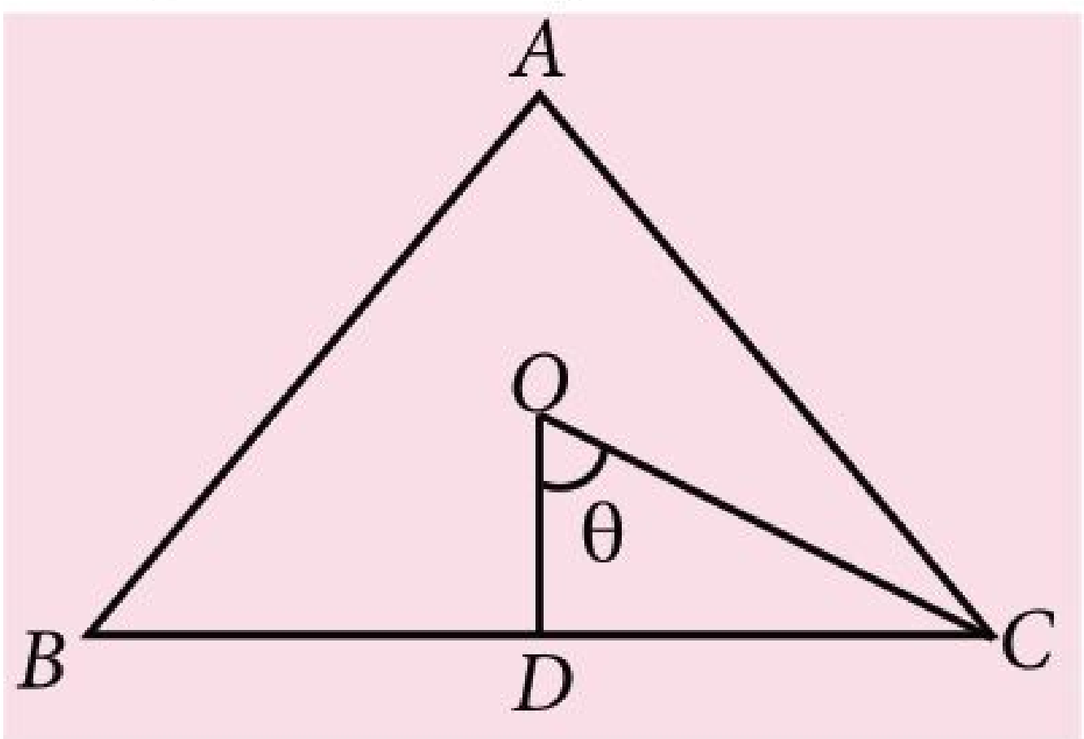
$\therefore$ $a^3 + b^3 = 8b^3\sin^3 10° + b^3 = b^3(2(4\sin^3 10°) + 1)$

$= b^3(2(3\sin 10° - \sin 30°)+1) = b^3(6\sin 10°)$

$= 3b^2(2b\sin 10°) = 3ac^2$

15. (b) : In the adjacent figure, we have $\angle OCB = \frac{\pi}{2} - A$

Let $\angle ODC = \pi - \left(\frac{\pi}{2} - A + \theta\right) = \frac{\pi}{2} + (A - \theta)$



If R be the circumradius of ΔOCD, then

$$\frac{OC}{\sin\left(\frac{\pi}{2} + (A-\theta)\right)} = 2R \ \Rightarrow \ 2R_1 = \frac{R}{\cos(A-\theta)}$$

As $D \to C, \ \theta \to 0 \ \therefore 2R_1 = \frac{R}{\cos A}$

16. (a, d) : $P = \frac{1}{2}\cos x[\sin(y+z) + \sin(y-z)]$

$$\geq \frac{1}{2}\cos x \cdot \sin(y+z) = \frac{1}{2}\cos^2 x$$

But $x = \frac{\pi}{2} - (y+z) \leq \frac{\pi}{2} - 2\cdot\frac{\pi}{12} = \frac{\pi}{3} \ \therefore \ P \geq \frac{1}{8}$

Again, $P=\frac{1}{2}\cos z[\sin(x+y)-\sin(x-y)]$

$$\Rightarrow P\le\frac{1}{2}\cos^2 z=\frac{1+\cos 2z}{4}\Rightarrow P\le\frac{2+\sqrt{3}}{8}$$

17. (c, d) : $$\sum_{m=1}^{6}\frac{\sin\left[\left(\theta+\frac{m\pi}{4}\right)-\left(\theta+\frac{(m-1)\pi}{4}\right)\right]}{\left[\sin\left(\theta+\frac{(m-1)\pi}{4}\right)\cdot\sin\left(\theta+\frac{m\pi}{4}\right)\right]}=4$$

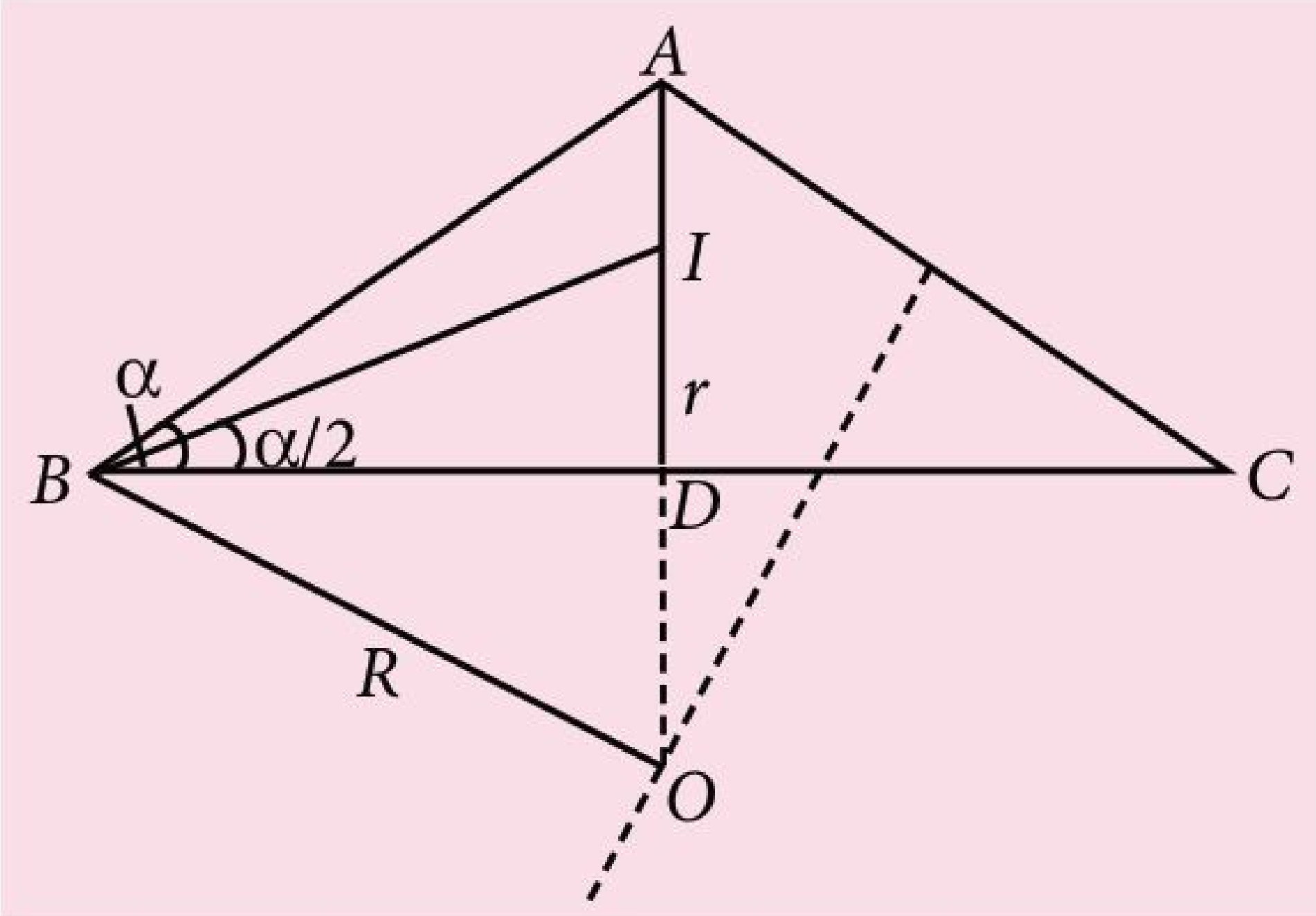
$$\Rightarrow\ \cot\theta-\cot\left(\theta+\frac{\pi}{4}\right)+\cot\left(\theta+\frac{\pi}{4}\right)-\cot\left(\theta+\frac{2\pi}{4}\right)+\ldots.$$

$$+\cot\left(\theta+\frac{5\pi}{4}\right)-\cot\left(\theta+\frac{6\pi}{4}\right)=4$$

$\Rightarrow\ \cot\theta+\tan\theta=4$

$\therefore\ \theta=\frac{\pi}{12}$ or $\frac{5\pi}{12}$

18. (a, c, d) :



Let ABC be the isosceles triangle with $AB=AC=b$ and $\angle B=\angle C=\alpha$. Let AD be the perpendicular bisector of the side BC. Since ΔABC is isosceles, AD is also the bisector of angle A, so that O and I both lie on AD. We have $OB=R$ and $ID=r$. Also, since O is the circumcentre, we get $OA=OB=R$. Therefore, from isosceles triangle OAB

$$\frac{OB}{\sin(90^\circ-\alpha)}=\frac{AB}{\sin 2\alpha}\quad\Rightarrow\quad R=\frac{b\cos\alpha}{2\sin\alpha\cos\alpha}$$

$$=\frac{1}{2}b\operatorname{cosec}\alpha$$

Again $\Delta=BD\cdot AD=b\cos\alpha\ b\sin\alpha$

$$=\frac{1}{2}b^2\sin 2\alpha$$

$$r=\frac{\Delta}{s}=\frac{\frac{1}{2}b^2\sin 2\alpha}{\frac{1}{2}(b+b+2b\cos\alpha)}=\frac{b\sin 2\alpha}{2(1+\cos\alpha)}$$

Further $OI=|OD+DI|=|OD+r|$

Because $\alpha<\frac{\pi}{4}, A>\frac{\pi}{2}$ and O lies on AD produced.

Now, from right-angled triangle ODB, we get

$OD^2=OB^2-BD^2=R^2-(b\cos\alpha)^2$

$$=\frac{1}{4}\frac{b^2}{\sin^2\alpha}-b^2\cos^2\alpha$$

$$=\frac{b^2(1-4\sin^2\alpha\cos^2\alpha)}{4\sin^2\alpha}$$

$$=\frac{b^2(\cos^2\alpha-\sin^2\alpha)^2}{4\sin^2\alpha}=\frac{b^2\cos^2 2\alpha}{(2\sin\alpha)^2}\ \text{[from (a)]}$$

$$\Rightarrow\quad OD=\frac{b\cos 2\alpha}{2\sin\alpha}$$

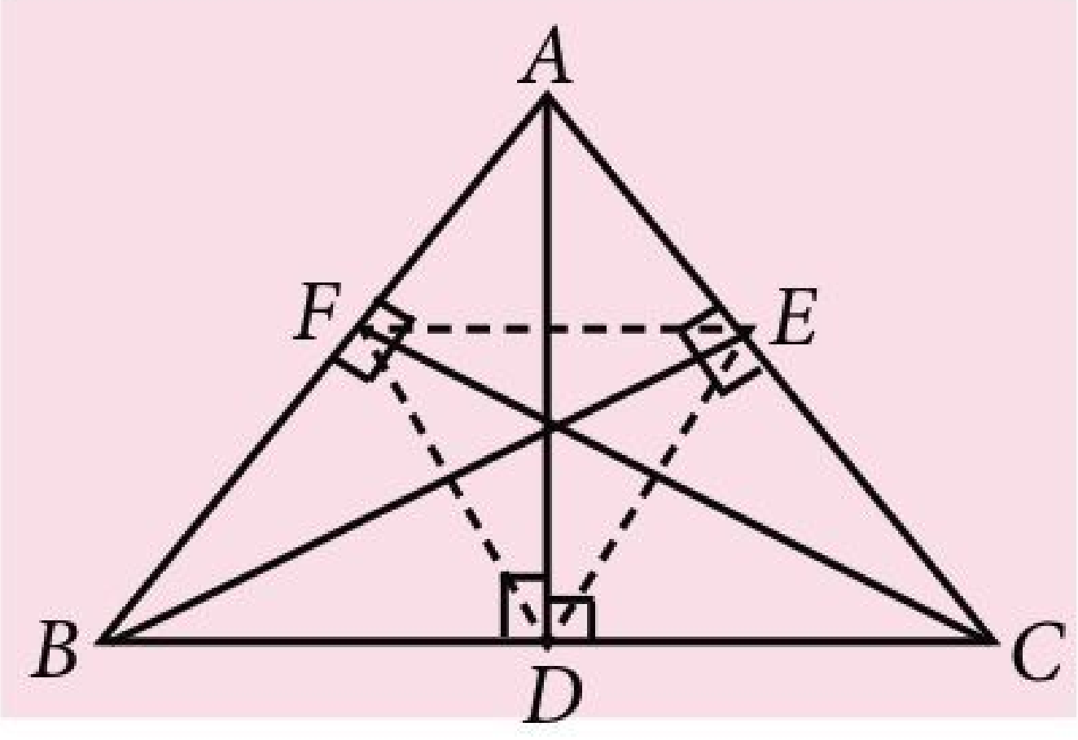
$$\therefore\quad OI=\left|\frac{b\sin 2\alpha}{2(1+\cos\alpha)}+\frac{b\cos 2\alpha}{2\sin\alpha}\right|$$

$$=\left|\frac{b\sin 2\alpha}{4\cos^2(\alpha/2)}+\frac{b\cos 2\alpha}{4\sin(\alpha/2)\cos(\alpha/2)}\right|$$

$$=\left|\frac{b}{4\cos(\alpha/2)}\cdot\left(\frac{\sin 2\alpha\sin(\alpha/2)+\cos 2\alpha\cos(\alpha/2)}{\sin(\alpha/2)\cos(\alpha/2)}\right)\right|$$

$$\therefore\quad OI=\left|\frac{b\cos(3\alpha/2)}{2\sin\alpha\cos(\alpha/2)}\right|$$

19. (b, c, d) : Here,



$EF=a\cos A$, $FD=b\cos B$,

$ED=c\cos C$

or $\quad EF+FD+ED$

$=R(\sin 2A+\sin 2B+\sin 2C)$

$=4R\sin A\sin B\sin C$

For a ΔABC, $\frac{\sin A+\sin B+\sin C}{3}\ge(\sin A\sin B\sin C)^{1/3}$

$$\Rightarrow\ \sin A\sin B\sin C\le\left(\frac{\sqrt{3}}{2}\right)^3=\frac{3\sqrt{3}}{8}$$

$\therefore\quad EF+FD+ED=4R\sin A\sin B\sin C$

$$\le\ \frac{3\sqrt{3}}{2}R=3\sqrt{3}r$$

But given $EF+FD+ED\ge 3\sqrt{3}r$, which is possible only when triangle is equilateral.

Also $r=(9-a)\tan\frac{A}{2}\ \Rightarrow\ \sqrt{3}=(9-a)\frac{1}{\sqrt{3}}$

$\Rightarrow\quad a=b=c=6$ (since triangle is equilateral)

Now, $\frac{\text{Area of }\Delta ABC}{\text{Area of }\Delta DEF}=\frac{\frac{1}{2}bc\sin A}{\frac{1}{2}b\cos B\ c\cos C\sin(\pi-2A)}$

$$=\frac{1}{2\cos A\cos B\cos C}=\frac{4}{1}$$

Also area of $\Delta ABC=\frac{1}{2}bc\sin A=\frac{36}{2}\frac{\sqrt{3}}{2}=9\sqrt{3}$ sq. units.

20. (b) : $\left(\frac{s(s-a)}{\Delta}\right)^2+4\left(\frac{s(s-b)}{\Delta}\right)^2+9\left(\frac{s(s-c)^2}{\Delta}\right)=\frac{36s^2}{49r^2}$

$\Rightarrow\ \frac{s^2}{\Delta^2}\left[(s-a)^2+4(s-b)^2+9(s-c)^2\right]=\frac{36s^2}{49}\frac{s^2}{\Delta^2}$

$\Rightarrow\ \left(\frac{s-a}{6}\right)^2+\left(\frac{s-b}{3}\right)^2+\left(\frac{s-c}{2}\right)^2=\frac{s^2}{49}$

Let $\frac{s-a}{6}=l,\frac{s-b}{3}=m,\frac{s-c}{2}=n\ \Rightarrow s=6l+3m+2n$

$\Rightarrow\ 49(l^2+m^2+n^2)=(6l+3m+2n)^2$

$\Rightarrow\ \frac{l}{6}=\frac{m}{3}=\frac{n}{2}=k$

$\therefore\ s-a=36k,\ s-b=9k,\ s-c=4k$

$\Rightarrow\ s=49k,\ a=13k,\ b=40k,\ c=45k$

$\therefore\ a:b:c=13:40:45$

21. (a) : $\cos c=\frac{13^2+40^2-45^2}{2\times13\times40}=\frac{-256}{2\times13\times40}=\frac{-16}{65}$

22. (1) : $[\sin x]=\frac{x}{2\pi}-\left[\frac{x}{2\pi}\right]+\frac{2x}{5\pi}-\left[\frac{2x}{5\pi}\right]$

$\therefore$ Number of solutions = 1

23. (5) : Put $\cot x=u$

$\therefore\ 3u^2+8u+3=0$

Both roots are real and product of roots = 1

But $\cot x$ is bijective in $(0,\pi)$. Let x_1, x_2 are roots such that $0<x_1, x_2<\pi$.

But $\cot x_1$, $\cot x_2$ are both negative.

$\therefore\ \frac{\pi}{2}<x_1,x_2<\pi$

But $\pi<x_1+x_2<2\pi$

$\therefore\ \cot x_1\cdot\cot x_2=1$

$\Rightarrow\ \cot x_1\cdot\cot\left(\frac{3\pi}{2}-x_1\right)=1\ \therefore\ x_1+x_2=\frac{3\pi}{2}$

Similarly $x_3+x_4=\frac{7\pi}{2}$

$\therefore\ k=5\pi$

24. (0) : ∵ R.H.S. is always odd.
While L.H.S. is always even.

25. (5) : ∵ $SI=r$, where s is circumcentre and I is incentre

$\Rightarrow\ R^2-2Rr-r^2=0$

$\Rightarrow\ \left(\frac{R}{r}\right)^2-2\left(\frac{R}{r}\right)^2-1=0$

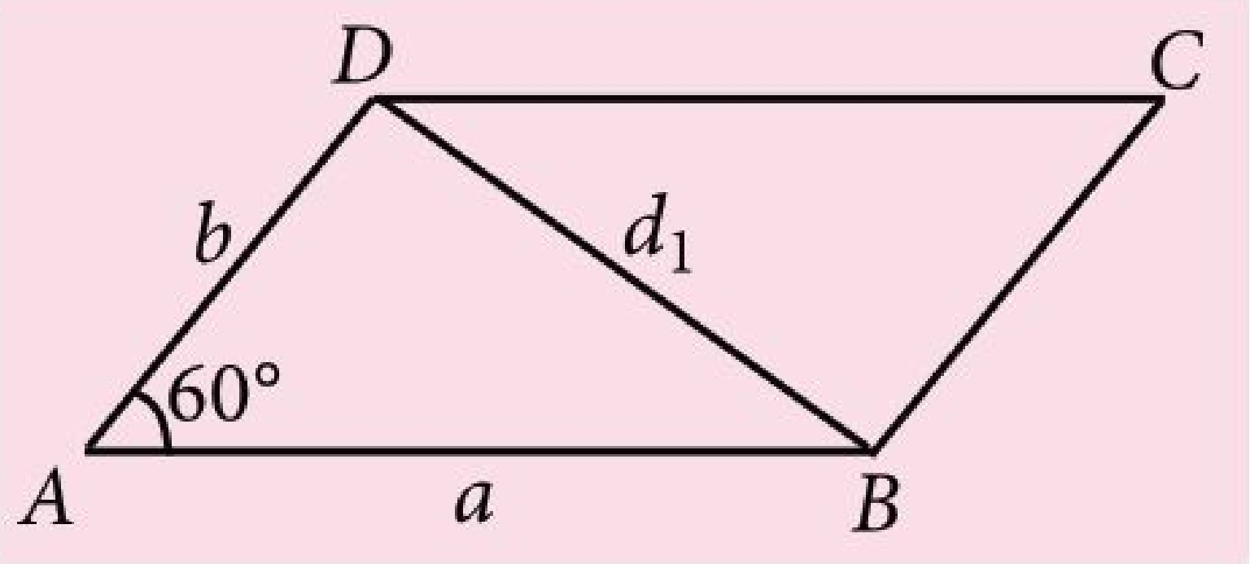
$\Rightarrow\ \frac{R}{r}=\frac{2\pm2\sqrt{2}}{2}=1\pm\sqrt{2}$

$\therefore\ \frac{R}{r}=\sqrt{2}+1$

$\therefore\ \cos A+\cos B+\cos C=1+\frac{r}{R}=1+\sqrt{2}-1=\sqrt{2}$

$\therefore\ [4(\cos A+\cos B+\cos C)]=[4\sqrt{2}]=5$

26. (1) : Applying cosines rule in ΔABD and ΔBCD , we get $2a^2+2b^2=d_1^2+d_2^2$, where d_1, d_2 are lengths of diagonals $(d_1<d_2)$.



Given $d_2^2=3d_1^2$

$\therefore\ a^2+b^2=2d_1^2$

also $ab=d_1^2$

$\Rightarrow\ a^2+b^2+2ab=3ab+d_1^2=4d_1^2$ ...(i)

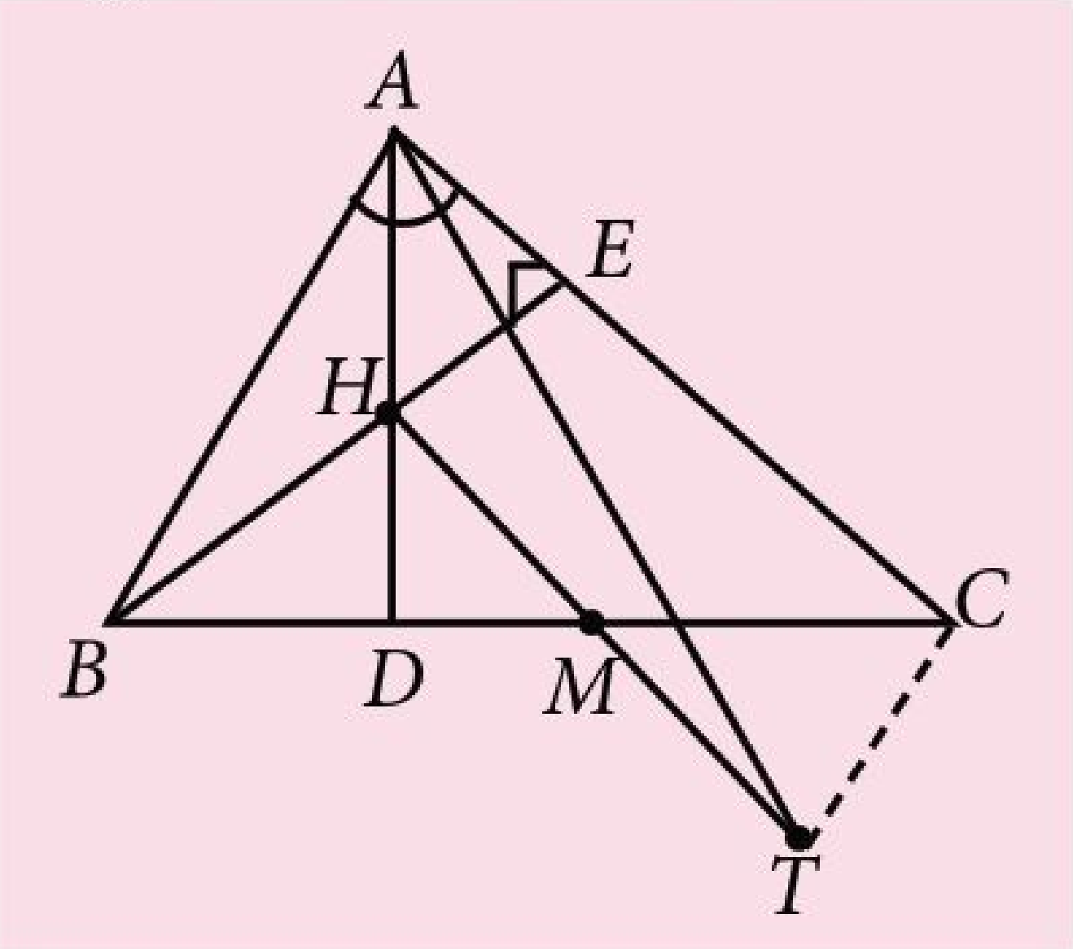
$\Rightarrow\ (a+b)^2=4d_1^2\ \Rightarrow\ a+b=2d_1$ ...(ii)

Using (i) and (ii), we get $a=b=d_1$ *i.e.*, $\frac{a}{b}=1$.

27. (8) : ΔBMH and ΔCMT are similar.

$\therefore\ \angle MBH=\angle MCT$

$\Rightarrow\ BH||CT,\ CT\perp AC$ and $CT=BH$



Now, $AT^2=TC^2+AC^2=BH^2+AC^2$

$=BD^2\operatorname{cosec}^2C+AD^2\operatorname{cosec}^2C$

$=\frac{AB^2}{\sin^2C}=\frac{a^2}{\sin^2A}=16\times4=64$

$\therefore\ AT=8$

◆ ◆

CBSE* warm-up!

Term-II

CLASS-XI

Series-1 Trigonometric Functions | Linear Inequalities | Permutations and Combinations

Time Allowed : 2 hours
Maximum Marks : 40

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 8 marks and Part-B carries 32 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 4 MCQs.*
3. *Section-II contains 1 case study-based questions.*

Part - B :

1. *It consists of four Sections-III, IV, V and VI.*
2. *Section-III comprises of 5 questions of 1 mark each.*
3. *Section-IV comprises of 4 questions of 2 marks each.*
4. *Section-V comprises of 3 questions of 3 marks each.*
5. *Section-VI comprises of 2 questions of 5 marks each.*
6. *Internal choice is provided in 1 question of Section-III, 1 question of Section-IV, 1 question of Section-V and 2 questions of section-VI. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. If $A + B = \dfrac{5\pi}{4}$, then value of $(1 + \tan A)(1 + \tan B)$ equals

(a) 1 (b) 2 (c) −2 (d) −1

2. If ${}^{10}C_{x-1} > 2\ {}^{10}C_x$, then set of values of x is

(a) $[1, 10]$ (b) $(7, \infty)$
(c) $\{1, 2, 3\}$ (d) $\{8, 9, 10\}$

3. If $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$ and $x \sin\theta = y \cos\theta$, then $x^2 + y^2$ is equal to

(a) 2 (b) 0 (c) 3 (d) 1

4. If $-8 \le 5x - 3 < 7$, then $x \in$

(a) $(-1, 2)$ (b) $[-1, 2)$ (c) $[-2, \infty)$ (d) $[-2, 0)$

Section - II

Case study-based question is compulsory. Attempt any 4 sub parts. Each sub-part carries 1 mark.

5. Amit's mother gave him ₹250 to buy some packet of rice and maggi from the market. The cost of one packet of rice is ₹50 and that of one packet of maggi is ₹25. Let x denotes the number of packet of rice and y denotes the number of packet of maggi.

**This is for practice purpose. CBSE has yet not released the official sample paper. So, the pattern is suggestive only. For latest information visit www.cbse.gov.in.*

Based on the above information, answer the following questions.

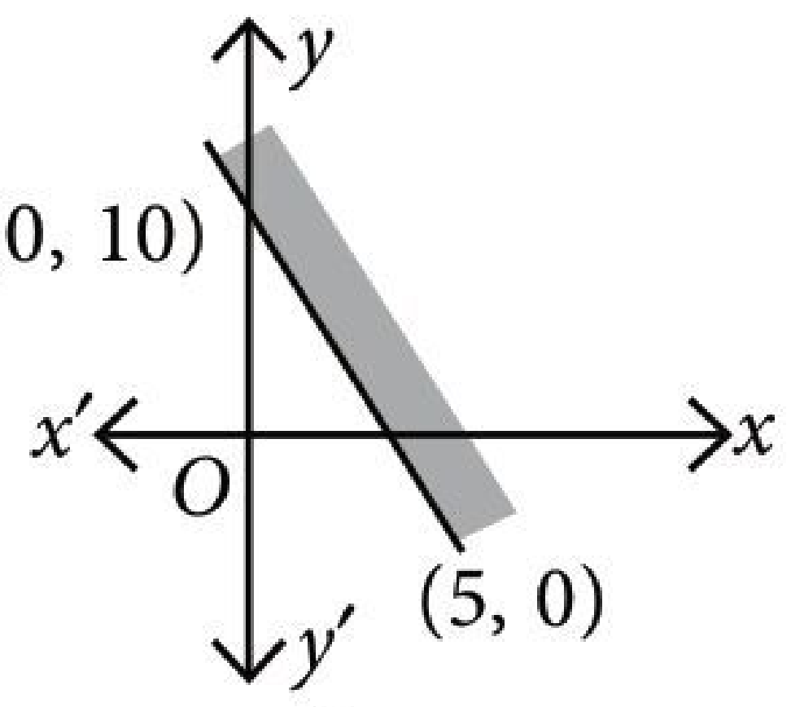
(i) The inequality that represents the given situation is
(a) $50x + 25y > 250$ (b) $50x + 25y \geq 250$
(c) $50x + 25y < 250$ (d) $50x + 25y \leq 250$

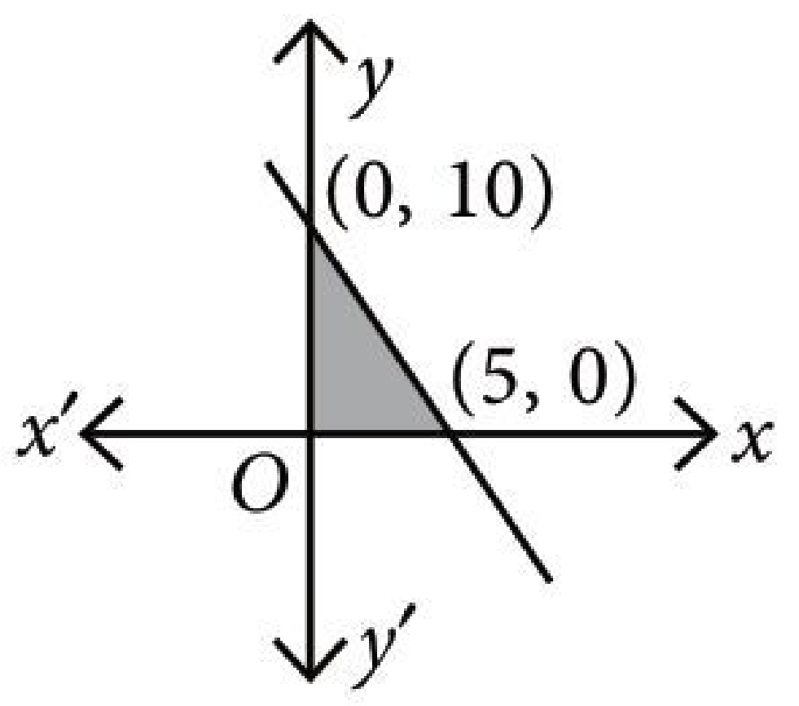
(ii) If he buys 3 packets of rice and 2 packets of maggi, then the amount that he paid is
(a) ₹110 (b) ₹120 (c) ₹200 (d) ₹140

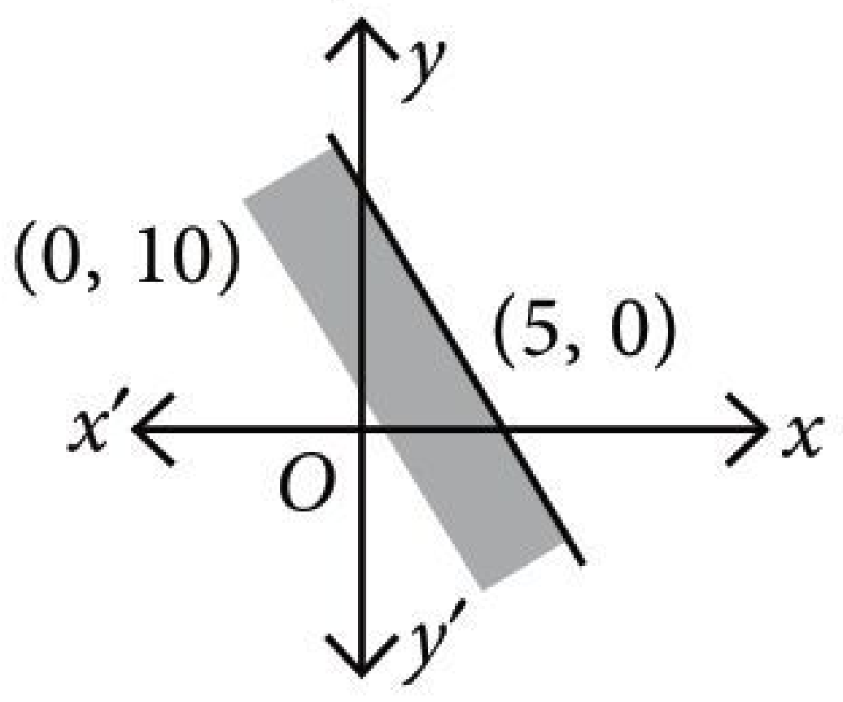
(iii) If he buys 4 packets of rice and spends entire amount of ₹250, then the maximum number of packets of maggi that he can buy is
(a) 3 (b) 2 (c) 5 (d) 6

(iv) The maximum value of x and y respectively, if he spends his entire amount is
(a) 5, 10 (b) 5, 6 (c) 4, 5 (d) 5, 5.

(v) The graph representing the given situation is

(a)


(b)


(c)


(d) None of these

PART - B

Section - III

6. Write $\frac{13\pi}{4}$ radians in the degrees.

7. How many different words can be formed by using all the letters of word 'SCHOOL'?

8. Find the value of $\sin\left(-\frac{11\pi}{3}\right)$.

OR

Express the following as a sum or difference:

$$2\sin\frac{5\theta}{2}\cos\frac{7\theta}{2}$$

9. Solve the inequation : $3x - 10 > 5x + 1$

10. Find the value of sin 15°.

Section - IV

11. If $5\sin x = 3$, then find the value of $\frac{\sec x - \tan x}{\sec x + \tan x}$.

12. In how many ways we can select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

OR

In how many ways 9 pictures can be hung from 6 picture nails on a wall?

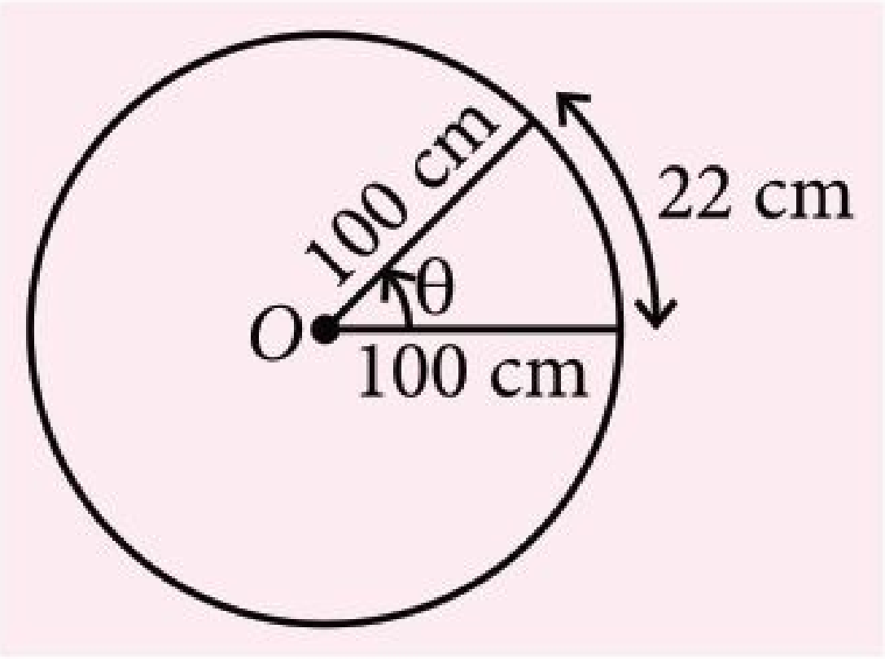
13. Solve the following inequation :

$$\frac{2x-3}{4} + 19 \geq 13 + \frac{4x}{3}$$

14. Prove that : $\sqrt{2+\sqrt{2+2\cos 4x}} = 2\cos x, 0 < x < \frac{\pi}{4}$

Section - V

15. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm as shown in figure. $\left(\text{Use } \pi = \frac{22}{7}\right)$.



16. **(i)** How many different words can be formed with the letters of the word HARYANA?

(ii) How many of these begin with H and end with N?

(iii) In how many of these H and N are together?

OR

How many words, with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word 'DAUGHTER'?

17. A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?

Section - VI

18. Prove that : $16\sin^5\theta - 20\sin^3\theta + 5\sin\theta = \sin 5\theta$

OR

Prove that :

$$\cos^2 x + \cos^2\left(x+\frac{\pi}{3}\right) + \cos^2\left(x-\frac{\pi}{3}\right) = \frac{3}{2}$$

19. A question paper contains 12 questions, divided into three parts. Part *A* contains 6 questions while

B and C contains 3 questions each. A candidate is required to attempt 6 questions selecting at least two from part A and at least one from each of part B and part C. In how many ways can the candidate select 6 questions?

OR

If ${}^nC_r : {}^nC_{r+1} = 1 : 2$ and ${}^nC_{r+1} : {}^nC_{r+2} = 2 : 3$, then determine the values of n and r.

SOLUTIONS

1. **(b) :** $\tan(A+B) = \tan\frac{5\pi}{4} = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4}$

$\therefore$ $\tan A + \tan B = 1 - \tan A \tan B$

$\Rightarrow$ $\tan A + \tan B + \tan A \tan B = 1$

$\Rightarrow$ $(1 + \tan A) + \tan B(1 + \tan A) = 1 + 1$

(By adding 1 on both sides)

$\Rightarrow$ $(1 + \tan A)(1 + \tan B) = 2$

2. **(d) :** $\frac{10!}{(x-1)!(11-x)!} > 2\cdot\frac{10!}{(10-x)!x!}$, $1 \le x \le 10$

$\Rightarrow \frac{1}{11-x} > 2\cdot\frac{1}{x} \Rightarrow x > 22 - 2x \Rightarrow x > \frac{22}{3}$.

Hence, $x = 8, 9, 10$.

3. **(d) :** $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ ...(i)

$x\sin\theta = y\cos\theta \Rightarrow y = x\frac{\sin\theta}{\cos\theta}$ ...(ii)

From (i) and (ii) we get,

$x\sin^3\theta + x\frac{\sin\theta}{\cos\theta}\cdot\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow x\sin^3\theta + x\sin\theta\cos^2\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$

Putting the value of x in (ii), we get $y = \sin\theta$

$\Rightarrow x^2 + y^2 = \sin^2\theta + \cos^2\theta = 1$

4. **(b) :** We have, $-8 \le 5x - 3 < 7$

$\Rightarrow -5 \le 5x < 10 \Rightarrow -1 \le x < 2 \therefore x \in [-1, 2)$

5. **(i) (d) :** Total amount = ₹250

Cost of one packet of rice = ₹50

and cost of one packet of maggi = ₹25

Since, x and y denote the number of packets of rice and maggi respectively,

Total amount spent by Amit is $50x + 25y$.

$\therefore$ Required inequality is $50x + 25y \le 250$.

(ii) (c) : Cost of 3 packet of rice = ₹(50×3) = ₹150

Cost of 2 packets of maggi = ₹ (25×2) = ₹50

Total amount paid = ₹$(150 + 50)$ = ₹200

(iii) (b) : If he spends his entire amount, then

We have, $50x + 25y = 250$ (1)

Since, number of packet of rice = 4

$\therefore$ At $x = 4$, equation (1) becomes

$50 \times 4 + 25y = 250$

$\Rightarrow 200 + 25y = 250 \Rightarrow 25y = 250 - 200$

$\Rightarrow 25y = 50 \Rightarrow y = 2$

$\therefore$ Maximum number of packets of maggi that he can buy is 2.

(iv) (a) : If he spends his entire amount, then we have, $50x + 25y = 250$

At $x = 0$, $y = 10$ and at $y = 0$, $x = 5$

$\therefore$ Maximum value of x and y which satisfies this equation is 5 and 10 respectively.

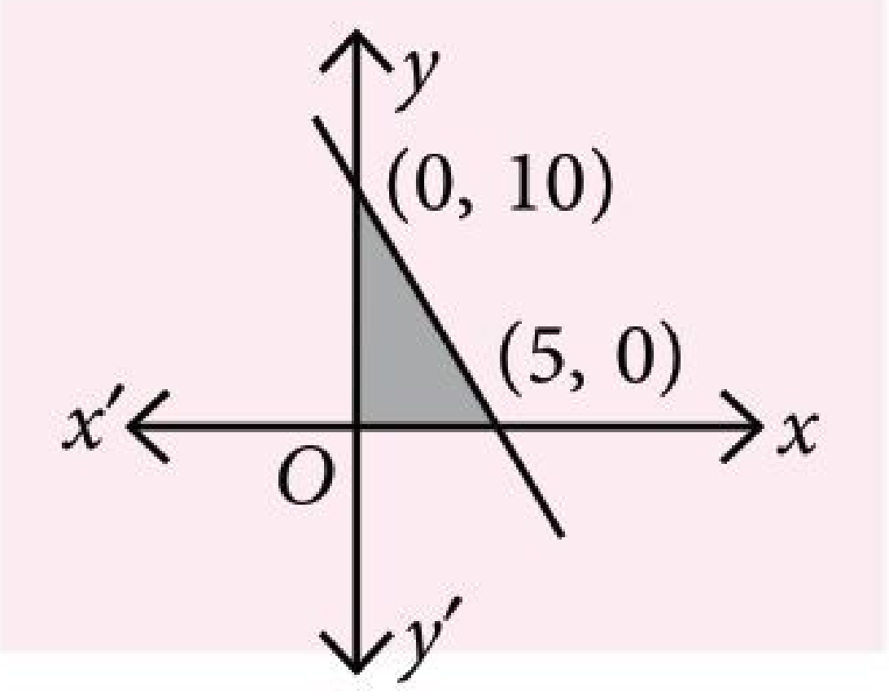
(v) (b) : As, x and y denotes the number of packets of rice and maggi respectively.

$\therefore$ $x \ge 0$ and $y \ge 0$.

and inequality representing the given situation is

$50x + 25y \le 250$.

$\therefore$ The graph showing these inequalities is given by



6. We have, $\frac{13\pi}{4}$ radians $= \left(\frac{13\pi}{4}\times\frac{180}{\pi}\right)^\circ = 585^\circ$

7. There are 6 letters (including 2 'O's in the word 'SCHOOL'.

$\therefore$ Total number of words formed $= \frac{6!}{2!} = 360$

8. We have, $\sin\left(-\frac{11\pi}{3}\right) = -\sin\left(\frac{11\pi}{3}\right) = -\sin\left(4\pi - \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

OR

We have, $2\sin\frac{5\theta}{2}\cos\frac{7\theta}{2} = \sin\left(\frac{5\theta}{2} + \frac{7\theta}{2}\right) + \sin\left(\frac{5\theta}{2} - \frac{7\theta}{2}\right)$

$[\because 2\sin A\cos B = \sin(A+B) + \sin(A-B)]$

$= \sin 6\theta + \sin(-\theta) = \sin 6\theta - \sin\theta\ [\because \sin(-\theta) = -\sin\theta]$

9. Given inequation is $3x - 10 > 5x + 1$

$\Rightarrow 3x - 5x > 1 + 10 \Rightarrow -2x > 11 \Rightarrow x < -\frac{11}{2}$

Hence, the solution set of the given inequation is $\left\{x \in \mathbf{R} : x < -\frac{11}{2}\right\}$; *i.e.*, $\left(-\infty, -\frac{11}{2}\right)$.

10. We have, sin 15° = sin (45° – 30°)

$$= \sin 45° \cos 30° - \cos 45° \sin 30°$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

11. We have, $5 \sin x = 3 \Rightarrow \sin x = \frac{3}{5}$

Since, $\sin^2 x + \cos^2 x = 1$

$$\Rightarrow \cos^2 x = 1 - \sin^2 x = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\Rightarrow \cos x = \pm\frac{4}{5}$$

$$\therefore \sec x = \pm\frac{5}{4} \text{ and } \tan x = \pm\left(\frac{3}{5}\times\frac{5}{4}\right) = \pm\frac{3}{4}$$

Hence, $\dfrac{\sec x - \tan x}{\sec x + \tan x} = \dfrac{\frac{5}{4}-\frac{3}{4}}{\frac{5}{4}+\frac{3}{4}} = \dfrac{2}{4}\times\dfrac{4}{8} = \dfrac{1}{4}$

12. Total players = 17, Bowlers = 5,
Non-bowlers = 12
Team of 11 players is to be selected having exactly 4 bowlers
∴ Required number of ways = ${}^5C_4 \times {}^{12}C_7 = 5 \times 792$
= 3960

OR

Required number of ways $= {}^9P_6 = \dfrac{9!}{(9-6)!}$

$$= \frac{9!}{3!} = 9 \times 8 \times 7 \times 6 \times 5 \times 4 = 60480$$

13. We have, $\dfrac{2x-3}{4} + 19 \geq 13 + \dfrac{4x}{3}$

$$\Rightarrow \frac{2x-3}{4} - \frac{4x}{3} \geq 13 - 19 \Rightarrow \frac{6x-9-16x}{12} \geq -6$$

$$\Rightarrow -10x - 9 \geq -72 \Rightarrow -10x \geq -72 + 9 \Rightarrow -10x \geq -63$$

$$\Rightarrow x \leq \frac{63}{10}$$

Thus, solution set of given inequation is $\left(-\infty, \dfrac{63}{10}\right]$.

14. L.H.S. = $\sqrt{2+\sqrt{2+2\cos 4x}}$

$$= \sqrt{2+\sqrt{2(1+\cos 4x)}} = \sqrt{2+\sqrt{2\cdot 2\cos^2 2x}}$$

$$= \sqrt{2+2\cos 2x} = \sqrt{2(1+\cos 2x)}$$

$$= \sqrt{2\cdot 2\cos^2 x} = 2\cos x = \text{R.H.S.}$$

15. We know that $l = r\theta$
where, l = Length of arc = 22 cm,
r = Radius of circle = 100 cm and
θ = Angle subtended at the centre

$$\therefore \theta = \frac{l}{r} = \frac{22}{100} = 0.22 \text{ radian}$$

$$= \left(0.22\times\frac{180}{\pi}\right)^\circ = \left(\frac{0.22\times 180\times 7}{22}\right)^\circ$$

$$= \left(\frac{22}{100}\times\frac{180\times 7}{22}\right)^\circ = \left(\frac{63}{5}\right)^\circ = \left(12\frac{3}{5}\right)^\circ$$

16. **(i)** There are 7 letters in the word 'HARYANA' of which 3 are A's and remaining all are each of its own kind.

So, total number of words $= \dfrac{7!}{3!} = 840$

(ii) After fixing H in first place and N in last place, we have 5 letters out of which three A's are alike.

So, total number of words $= \dfrac{5!}{3!} = 20$

(iii) Considering H and N together, we have, 7–2 + 1 = 6 letters out of which three A's are alike. These six letters can be arranged in $\dfrac{6!}{3!}$ ways. But H and N can be arranged among themselves in 2! ways.

∴ Required number of words $= \dfrac{6!}{3!}\times 2! = 240$

OR

The word DAUGHTER consists of 3 vowels and 5 consonants.
2 vowels and 3 consonants may be selected in ${}^3C_2 \times {}^5C_3 = 30$ ways
Also, 5 letters can be arranged in 5! ways *i.e.*, in 5 × 4 × 3 × 2 × 1 = 120 ways
∴ Number of words formed by 2 vowels and 3 consonants from the word 'DAUGHTER'
= 30 × 120 = 3600

17. Let x L of 30% acid solution is added to 600 L of 12% solution of acid.
According to given condition,

$$\frac{15}{100}(600+x) \leq \frac{12}{100}\times 600 + \frac{30}{100}x \leq \frac{18}{100}(600+x)$$

$\Rightarrow 15(600 + x) \leq 7200 + 30x \leq 18(600 + x)$
Consider, $15(600 + x) \leq 7200 + 30x$
$\Rightarrow 9000 + 15x \leq 7200 + 30x$
$\Rightarrow 1800 \leq 15x \Rightarrow x \geq 120$ ...(i)
Consider, $7200 + 30x \leq 18(600 + x)$
$\Rightarrow 7200 + 30x \leq 10800 + 18x$
$\Rightarrow 12x \leq 3600 \Rightarrow x \leq 300$ ...(ii)
From (i) and (ii), we get
$120 \leq x \leq 300$

Hence 30% acid solution should be added between 120 L and 300 L to get resulting mixture between 15% acid solution to 18% acid solution.

18. L.H.S. $= 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$

$= 16\sin^5\theta - 16\sin^3\theta - 4\sin^3\theta + 5\sin\theta + 3\sin\theta - 3\sin\theta$

$= 16\sin^3\theta[\sin^2\theta - 1] + \sin 3\theta + 2\sin\theta$

$= -16\sin^3\theta\cos^2\theta + \sin 3\theta + 2\sin\theta$

$= -(2\sin\theta\cos\theta)^2 \cdot 4\sin\theta + \sin 3\theta + 2\sin\theta$

$= -4\sin^2 2\theta\sin\theta + \sin 3\theta + 2\sin\theta$

$= 2\sin\theta[1 - 2\sin^2 2\theta] + \sin 3\theta$

$= 2\sin\theta\cos 4\theta + \sin 3\theta$

$= \sin 5\theta + \sin(-3\theta) + \sin 3\theta = \sin 5\theta =$ R.H.S.

OR

$$\text{L.H.S.} = \cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right)$$

$$= \frac{1}{2}\left[\cos 2x + 1 + \cos\left(2x + \frac{2\pi}{3}\right) + 1 + \cos\left(2x - \frac{2\pi}{3}\right) + 1\right]$$

$$= \frac{1}{2}\left[3 + \cos 2x + 2\cos 2x\cos\left(\frac{2\pi}{3}\right)\right]$$

$[\because \cos(A - B) + \cos(A + B) = 2\cos A\cos B]$

$$= \frac{1}{2}\left[3 + \cos 2x + 2\cos 2x\cos\left(\pi - \frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3 + \cos 2x + 2\cos 2x\left(-\cos\frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3 + \cos 2x + 2\cos 2x\left(\frac{-1}{2}\right)\right]$$

$$= \frac{1}{2}[3 + \cos 2x - \cos 2x] = \frac{3}{2} = \text{R.H.S.}$$

19. A candidate can select 6 questions in the following 6 ways :

(i) 2 from part A, 1 from part B and 3 from part C.
(ii) 2 from part A, 2 from part B and 2 from part C.
(iii) 2 from part A, 3 from part B and 1 from part C.
(iv) 3 from part A, 1 from part B and 2 from part C.
(v) 3 from part A, 2 from part B and 1 from part C.
(vi) 4 from part A, 1 from part B and 1 from part C.

$\therefore$ Required no. of ways $= C(6, 2)\cdot C(3, 1)\cdot C(3, 3) + C(6, 2)\cdot C(3, 2)\cdot C(3, 2) + C(6, 2)\cdot C(3, 3)\cdot C(3, 1) + C(6, 3)\cdot C(3, 1)\cdot C(3, 2) + C(6, 3)\cdot C(3, 2)\cdot C(3, 1) + C(6, 4)\cdot C(3, 1)\cdot C(3, 1)$

$$= \frac{6\times 5}{2}\times\frac{3}{1}\times 1 + \frac{6\times 5}{2}\times\frac{3}{1}\times\frac{3}{1} + \frac{6\times 5}{2}\times 1\times 3 + \frac{6\times 5\times 4}{6}\times\frac{3}{1}\times\frac{3}{1} + \frac{6\times 5\times 4}{6}\times\frac{3}{1}\times\frac{3}{1} + \frac{6\times 5}{2}\times\frac{3}{1}\times\frac{3}{1}$$

$= 45 + 135 + 45 + 180 + 180 + 135 = 720$

OR

${}^nC_r : {}^nC_{r+1} = 1 : 2$ and ${}^nC_{r+1} : {}^nC_{r+2} = 2 : 3$

$$\text{Now, } \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{1}{2} \Rightarrow \frac{\dfrac{n!}{r!(n-r)!}}{\dfrac{n!}{(r+1)!(n-r-1)!}} = \frac{1}{2}$$

$$\Rightarrow \frac{(r+1)!(n-r-1)!}{r!(n-r)!} = \frac{1}{2} \Rightarrow \frac{(r+1)}{(n-r)} = \frac{1}{2}$$

$\Rightarrow 2r + 2 = n - r \Rightarrow n = 3r + 2$ ...(i)

$$\text{Similarly, } \frac{{}^nC_{r+1}}{{}^nC_{r+2}} = \frac{2}{3} \Rightarrow \frac{\dfrac{n!}{(r+1)!(n-r-1)!}}{\dfrac{n!}{(r+2)!(n-r-2)!}} = \frac{2}{3}$$

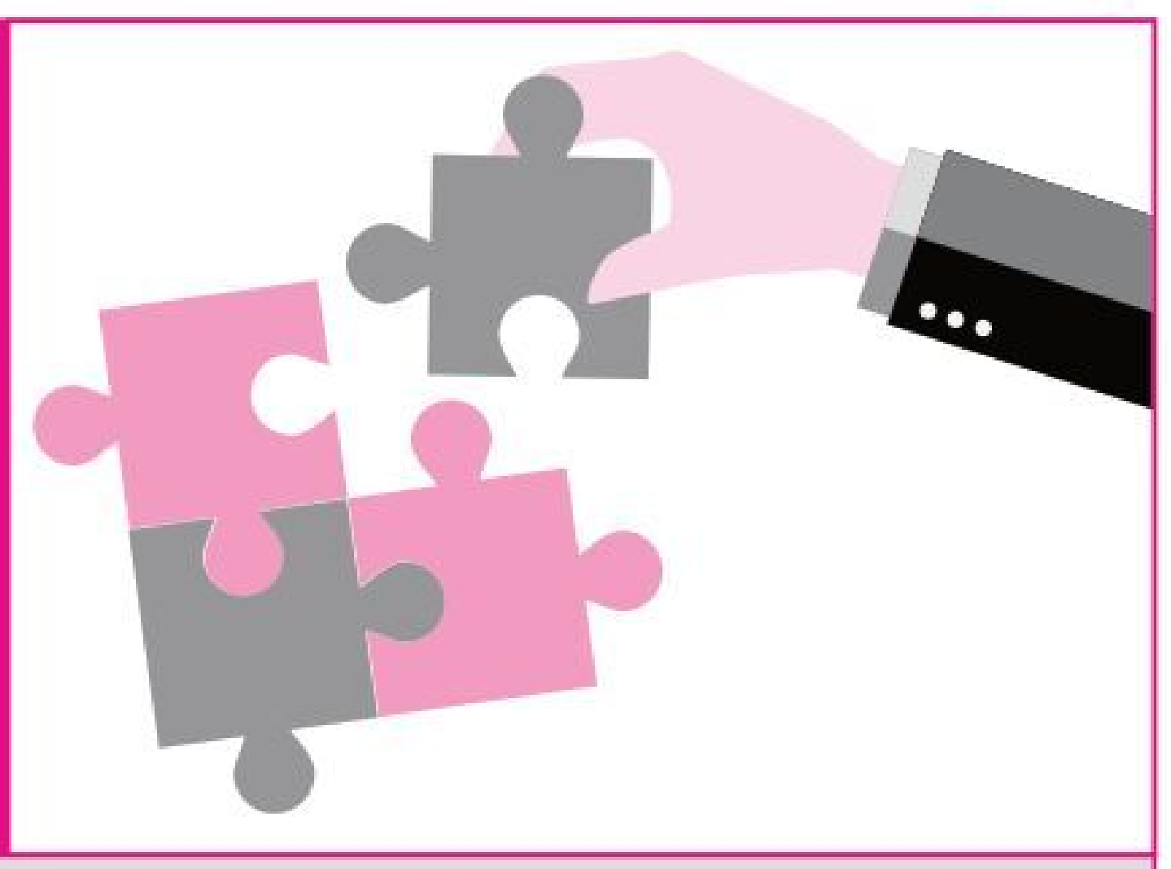
$$\Rightarrow \frac{(r+2)!(n-r-2)!}{(r+1)!(n-r-1)!} = \frac{2}{3} \Rightarrow \frac{r+2}{n-r-1} = \frac{2}{3}$$

$\Rightarrow 3r + 6 = 2n - 2r - 2 \Rightarrow 2n = 5r + 8$ ...(ii)

Solving (i) and (ii), we get $r = 4$ and $n = 14$

◆

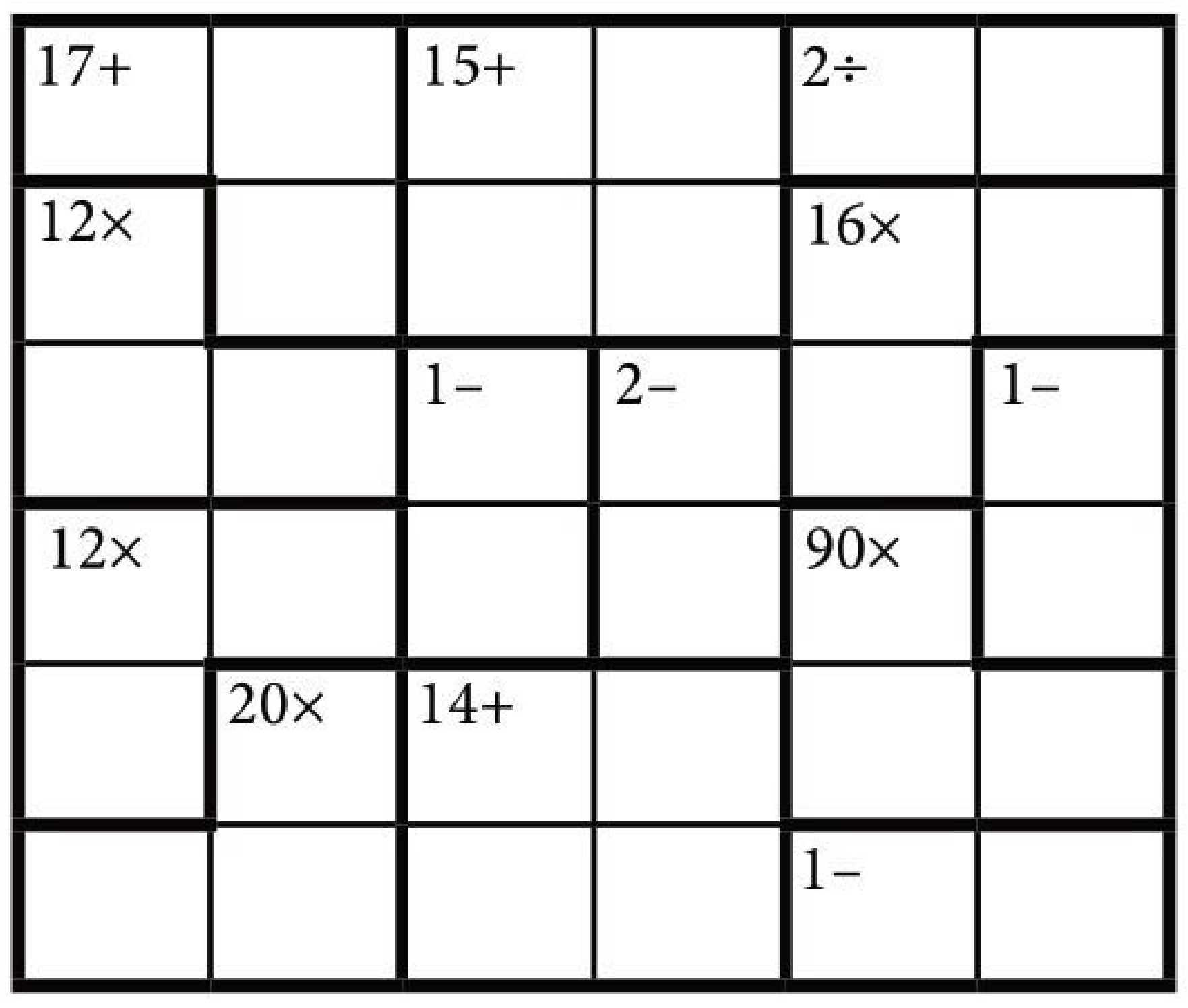
PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



17+		15+		2÷	
12×				16×	
		1–	2–		1–
12×				90×	
	20×	14+			
				1–	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' names with their valuable feedback will be published in next issue.

MONTHLY TEST
DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 7 : Limits and Derivatives

Time Taken : 60 Min.

Only One Option Correct Type

1. $\lim\limits_{n\to\infty}\dfrac{(1+2+3+\ldots+n)(1^3+2^3+3^3+\ldots+n^3)}{(1^2+2^2+3^2+\ldots+n^2)^2}=$

(a) $\dfrac{3}{8}$ (b) $\dfrac{5}{8}$ (c) $\dfrac{7}{8}$ (d) $\dfrac{9}{8}$

2. Let $f(x)$ is twice differentiable function and $f''(0)=2$, then $\underset{x\to 0}{\text{Lt}}\dfrac{2f(x)-3f(2x)+f(4x)}{x^2}$ equals

(a) 6 (b) 3 (c) 12 (d) 0

3. $\underset{x\to 0}{\text{Lt}}\dfrac{\sin^{-1}x-\tan^{-1}x}{x^3}$ equals, $|x|\le 1$

(a) 1 (b) –1 (c) 2 (d) 1/2

4. $\underset{n\to\infty}{\text{Lt}}\dfrac{2^3-1^3}{2^3+1^3}\cdot\dfrac{3^3-1^3}{3^3+1^3}\cdot\ldots\ldots\dfrac{n^3-1}{n^3+1}$ equals

(a) 1/3 (b) 2/3 (c) 1 (d) 3/2

5. Let $f(x)$ be a differentiable function and $f'(4)=5$. Then $\lim\limits_{x\to 2}\dfrac{f(4)-f(x^2)}{x-2}$ equals

(a) 0 (b) 5 (c) 20 (d) –20

6. If $y=\dfrac{\sin(x+9)}{\cos x}$, then $\dfrac{dy}{dx}$ at $x=0$

(a) cos 9 (b) sin 9 (c) 0 (d) 1

One or More Than One Option(s) Correct Type

7. If $L=\lim\limits_{x\to 0}\dfrac{1}{x^3}\left(\dfrac{1}{\sqrt{1+x}}-\dfrac{1+ax}{1+bx}\right)$ exists, then

(a) $a=\dfrac{1}{4}$ (b) $b=\dfrac{3}{4}$ (c) $L=-\dfrac{1}{32}$ (d) $L=\dfrac{1}{32}$

8. Which of the following is/are true?

(a) The derivative of $\sin 2x$ is $2(\cos^2x-\sin^2x)$.

(b) The derivative of $\dfrac{x^5-\cos x}{\sin x}$ is $\dfrac{x^5\cos x+5x^4\sin x+1}{(\sin x)^2}$.

SAMURAI SUDOKU

SOLUTION - DECEMBER 2021

1	6	9	5	2	3	7	4	8				6	2	8	5	9	3	4	7	1
8	5	4	1	9	7	3	6	2				1	4	7	6	2	8	9	5	3
2	7	3	4	8	6	9	1	5				9	3	5	7	1	4	6	2	8
3	8	6	7	1	5	4	2	9				7	8	9	1	5	2	3	4	6
4	9	7	2	3	8	6	5	1				4	5	2	8	3	6	7	1	9
5	2	1	9	6	4	8	7	3				3	6	1	4	7	9	2	8	5
7	1	2	8	4	9	5	3	6	7	1	8	2	9	4	3	8	5	1	6	7
6	4	8	3	5	1	2	9	7	3	4	5	8	1	6	9	4	7	5	3	2
9	3	5	6	7	2	1	8	4	6	2	9	5	7	3	2	6	1	8	9	4
						8	7	2	9	5	4	6	3	1						
						9	1	5	8	3	6	7	4	2						
						4	6	3	1	7	2	9	5	8						
6	1	3	5	4	8	7	2	9	4	8	3	1	6	5	8	3	7	4	2	9
5	2	7	1	3	9	6	4	8	5	9	1	3	2	7	1	4	9	5	6	8
4	8	9	7	2	6	3	5	1	2	6	7	4	8	9	6	5	2	3	7	1
1	7	4	9	8	5	2	6	3				7	9	3	4	2	1	8	5	6
2	6	8	3	1	7	5	9	4				2	1	4	5	6	8	9	3	7
9	3	5	4	6	2	8	1	7				6	5	8	7	9	3	2	1	4
8	9	2	6	7	4	1	3	5				8	4	6	3	7	5	1	9	2
7	4	1	2	5	3	9	8	6				5	7	2	9	1	4	6	8	3
3	5	6	8	9	1	4	7	2				9	3	1	2	8	6	7	4	5

(c) The derivative of $y = (5x^3 + 3x - 1)(x - 1)$ is $20x^3 - 15x^2 + 6x - 4$.

(d) The derivative of $\frac{x+\cos x}{\tan x}$ is $\frac{(1-\sin x)\sec^2 x-(x+\cos x)\tan x}{(\tan x)^2}$.

9. $\underset{x\to\frac{\pi}{4}}{\text{Lt}}\ (\tan x)^{\tan 2x}$ equals

(a) e^1 (b) e^2 (c) e^{-1} (d) e^{-2}

10. $\underset{x\to 0}{\text{Lt}}\ \sin^{-1}(\sec x)$ cannot be equal to

(a) $\pi/2$ (b) $-\pi/2$ (c) $3\pi/2$ (d) $\frac{-3\pi}{2}$

11. Let $f(x) = (ax + b)\cos x + (cx + d)\sin x$ and $f'(x) = x\cos x$ for all x, then

(a) $a = 0, b = c = 1, d = 0$
(b) $a = b = c = d = 1$
(c) $a = d = 0, b = c = -1$ (d) none of these

12. Which of the following is/are unity? (where [·] and {·} denote the greatest and fractional part functions, respectively)

(a) $\lim\limits_{x\to\infty}\sin\left(\frac{x^2+2}{x^2+1}\right)$ (b) $\lim\limits_{x\to\infty}\left[\frac{x^2+2}{x^2+1}\right]$

(c) $\lim\limits_{x\to\infty}\left\{\frac{x^2+2}{x^2+1}\right\}$ (d) $\lim\limits_{x\to\infty}\left\{\frac{x^2}{x^2+1}\right\}$

Comprehension Type

Paragraph for Q. No. 13 to 15

Let $f(x)$ be a cubic polynomial and $g(x)$ be a quadratic polynomial with same leading coefficients. Also, let $g(x) = 0$ has roots α and β. If $\lim\limits_{x\to\alpha}\frac{f(x)}{g(x)}=1$ and $\lim\limits_{x\to\beta}\frac{f(x)}{g(x)}=4$, then

13. The value of $\beta - \alpha$ is

(a) 1 (b) 2 (c) 3 (d) 4

14. $\lim\limits_{x\to\alpha}(1+g(x))^{\frac{1}{f(x)}} =$

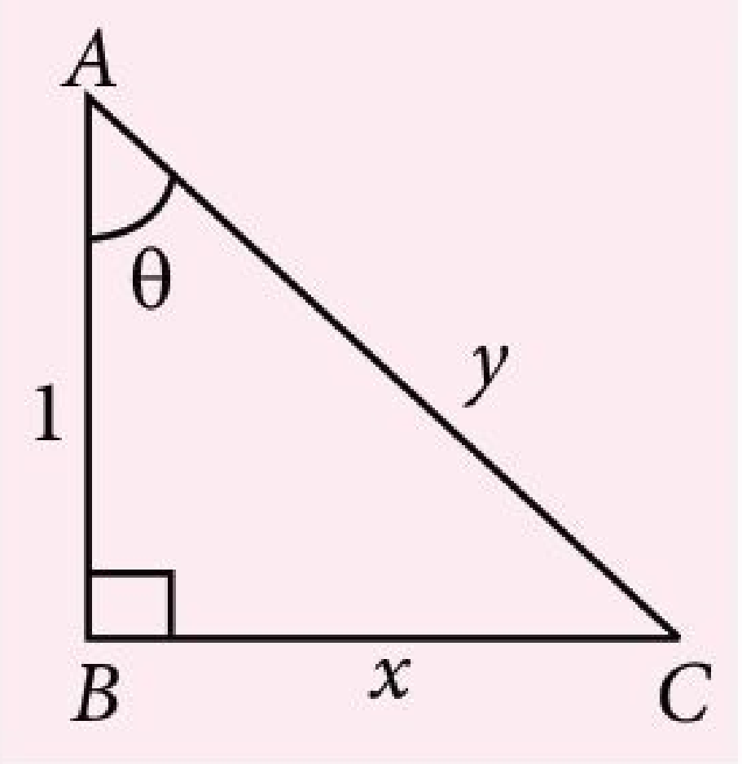
(a) 1 (b) e (c) e^2 (d) $1/e$

15. If $\lim\limits_{x\to\beta}(1+f(x))^{1/g(x)} = e^n$, then value of n is

(a) 1 (b) 0 (c) 2 (d) 4

Matrix Match Type

16. A right angled triangle ABC has legs of length 1 and x. The hypotenuse is y and the angle opposite to the side x is θ.



Match the Following :

Column-I		Column-II	
P.	$\lim\limits_{\theta\to\frac{\pi}{2}}\sqrt{y}-\sqrt{x}=$	1.	0
Q.	$\lim\limits_{\theta\to\frac{\pi}{2}} y-x=$	2.	$\frac{1}{2}$
R.	$\lim\limits_{\theta\to\frac{\pi}{2}} y^2-x^2=$	3.	1
S.	$\lim\limits_{\theta\to\frac{\pi}{2}} y^3-x^3=$	4.	∞

	P	Q	R	S
(a)	1	4	3	2
(b)	1	1	3	4
(c)	1	4	2	3
(d)	2	3	4	1

Numerical Answer Type

17. If $f(x) = 1+x+\frac{x^2}{2}+\ldots+\frac{x^{100}}{100}$, then sum of digits of $f'(1)$ is equal to __________.

18. If $\lim\limits_{x\to 0}\frac{1+a\cos x+b}{x^2} = 1$, then $b - a =$ __________.

19. If $f(x)=\frac{x-4}{2\sqrt{x}}$, then $f'(1)$ is equal to __________.

20. If $f(x)\begin{cases} x+2, & x\le -1 \\ cx^2 & x>-1 \end{cases}$, find c, if $\lim\limits_{x\to -1} f(x)$ exists.

◆ ◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

18+

WHAT IS
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CBSE TERM-I

Held on
6th Dec

SOLVED PAPER 2021

CLASS
12

We are happy to inform our readers that out of the 50 questions asked in CBSE Board Term-I Exam, 2021 more than 90% questions were either exactly same or of similar type from the ***MTG 100 Percent Exam Ready Term-I*** *&* ***NCERT at your Fingertips****, Mathematics Class-12.*

The references of few questions of paper having code : 065/1/4 are given here :

Exam Q. No.	MTG Books	Q. No.	P. No.
5	Exam Ready Term-I	24	28
9	Exam Ready Term-I	4	43
16	Exam Ready Term-I	46	80
17	Exam Ready Term-I	24	79
22	Exam Ready Term-I	24	4
23	NCERT at your Fingertips	33	5
24	NCERT at your Fingertips	4	23
25	NCERT at your Fingertips	18	23

Exam Q. No.	MTG Books	Q. No.	P. No.
26	NCERT at your Fingertips	46	35
31	Exam Ready Term-I	51	64
32	NCERT at your Fingertips	2	74
34	NCERT at your Fingertips	7	91
35	Exam Ready Term-I	28	79
37	Exam Ready Term-I	33	96
39	Exam Ready Term-I	26	96

and more such questions……

Time Allowed : 90 minutes
Maximum Marks : 40

General Instructions :

(i) This question paper contains 50 questions out of which 40 questions are to be attempted. All questions carry equal marks.
(ii) This question paper consists of three sections - Section A, B and C.
(iii) Section-A contains 20 questions. Attempt any 16 questions from Q. no. 1 to 20.
(iv) Section-B contains 20 questions. Attempt any 16 questions from Q. no. 21 to 40.
(v) Section-C contains 10 questions. Attempt any 8 questions from Q. no. 41 to 50.
(vi) There is only one correct option for every multiple choice question (MCQ). Marks will not be awarded for answering more than one option.
(vii) There is no negative marking.

SECTION-A

In this section, attempt any 16 questions out of Q. No. 1 to 20.

1. A relation R is defined on N. Which of the following is the reflexive relation?

(a) $R = \{(x, y): x > y, x, y \in N\}$
(b) $R = \{(x, y): x + y = 10, x, y \in N\}$
(c) $R = \{(x, y): xy$ is the square number, $x, y \in N\}$
(d) $R = \{(x, y): x + 4y = 10, x, y \in N\}$

2. The function $f : R \to R$ defined by $f(x) = 4 + 3\cos x$ is

(a) bijective
(b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto

3. If $y = \cot^{-1}x$, $x < 0$, then

(a) $\frac{\pi}{2} < y \le \pi$ (b) $\frac{\pi}{2} < y < \pi$
(c) $-\frac{\pi}{2} < y < 0$ (d) $-\frac{\pi}{2} \le y < 0$

4. The number of functions defined from $[1, 2, 3, 4, 5] \to \{a, b\}$ which are one-one is

(a) 5 (b) 3 (c) 2 (d) 0

5. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then $(A - 2I)(A - 3I)$ is equal to

(a) A (b) I (c) $5I$ (d) O

6. If P is a 3×3 matrix such that $P' = 2P + I$, where P' is the transpose of P, then

(a) $P = I$ (b) $P = -I$
(c) $P = 2I$ (d) $P = -2I$

7. If order of matrix A is 2×3, of matrix B is 3×2, and of matrix C is 3×3, then which one of the following is not defined?
(a) $C(A + B')$ (b) $C(A + B')'$
(c) BAC (d) $CB + A'$

8. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 27$, then the value of α is
(a) ± 1 (b) ± 2 (c) $\pm\sqrt{5}$ (d) $\pm\sqrt{7}$

9. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then the value of x is
(a) 3 (b) 5 (c) 7 (d) 9

10. The inverse of $\begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}$ is
(a) $\begin{bmatrix} -5 & 3 \\ 7 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$
(c) $\begin{bmatrix} -5 & 7 \\ 3 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -3 \\ -7 & -4 \end{bmatrix}$

11. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 59 & 69 & -1 \end{bmatrix}$, then A^{-1}
(a) is A (b) is $(-A)$
(c) is A^2 (d) does not exist

12. If the function $f(x) = \begin{cases} 3x - 8, & \text{if } x \le 5 \\ 2k, & \text{if } x > 5 \end{cases}$ is continuous, then the value of k is
(a) 2/7 (b) 7/2 (c) 3/7 (d) 4/7

13. The function $f(x) = [x]$, where $[x]$ is the greatest integer function that is less than or equal to x, is continuous at
(a) 4 (b) −2 (c) 1.5 (d) 1

14. If $y = \tan^{-1}(e^{2x})$, then $\frac{dy}{dx}$ is equal to
(a) $\frac{2e^{2x}}{1+e^{4x}}$ (b) $\frac{1}{1+e^{4x}}$
(c) $\frac{2}{e^{2x}+e^{-2x}}$ (d) $\frac{1}{e^{2x}-e^{-2x}}$

15. If $y^2(2 - x) = x^3$, then $\left(\frac{dy}{dx}\right)_{(1,1)}$ is equal to
(a) 2 (b) −2 (c) 3 (d) −3/2

16. The angle between the tangents to the curve $y = x^2 - 5x + 6$ at the point (2, 0) and (3, 0) is
(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/4$ (d) 0

17. The interval, in which function $y = x^3 + 6x^2 + 6$ is increasing, is
(a) $(-\infty, -4) \cup (0, \infty)$ (b) $(-\infty, -4)$
(c) $(-4, 0)$ (d) $(-\infty, 0) \cup (4, \infty)$

18. The value of x for which $(x - x^2)$ is maximum, is
(a) 3/4 (b) 1/2 (c) 1/3 (d) 1/4

19. If the corner points of the feasible region of an LPP are (0, 3), (3, 2) and (0, 5), then the minimum value of $Z = 11x + 7y$ is
(a) 21 (b) 33 (c) 14 (d) 35

20. The number of solutions of the system of inequations $x + 2y \le 3$, $3x + 4y \ge 12$, $x \ge 0$, $y \ge 1$ is
(a) 0 (b) 2 (c) finite (d) infinite

SECTION-B

In this section, attempt any 16 questions out of Q. No. 21 to 40.

21. The number of equivalence relations in the set {1, 2, 3} containing the elements (1, 2) and (2, 1) is
(a) 0 (b) 1 (c) 2 (d) 3

22. Let $f : R \to R$ be defined by $f(x) = 1/x$, for all $x \in R$, Then, f is
(a) one-one (b) onto
(c) bijective (d) not defined

23. The function $f : N \to N$ is defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

The function f is
(a) bijective (b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto

24. The value of $\sin^{-1}\left(\cos\frac{13\pi}{5}\right)$ is
(a) $-\frac{3\pi}{5}$ (b) $-\frac{\pi}{10}$ (c) $\frac{3\pi}{5}$ (d) $\frac{\pi}{10}$

25. If $\sin^{-1}x > \cos^{-1}x$. then x should lie in the interval
(a) $\left(-1, -\frac{1}{\sqrt{2}}\right)$ (b) $\left(0, \frac{1}{\sqrt{2}}\right)$
(c) $\left(\frac{1}{\sqrt{2}}, 1\right)$ (d) $\left(-\frac{1}{\sqrt{2}}, 0\right)$

26. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A' = I$, then the value of α is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{3\pi}{2}$

27. The determinant $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$ is equal to

(a) $k(3y + k^2)$ (b) $3y + k^3$
(c) $3y + k^2$ (d) $k^2(3y + k)$

28. If $A = \begin{bmatrix} 1 & -2 & 4 \\ 2 & -1 & 3 \\ 4 & 2 & 0 \end{bmatrix}$ is the adjoint of a square matrix B, then B^{-1} is equal to

(a) $\pm A$ (b) $\pm\sqrt{2}A$ (c) $\pm\frac{1}{\sqrt{2}}B$ (d) $\pm\frac{1}{\sqrt{2}}A$

29. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, then $A^5 - A^4 - A^3 + A^2$ is equal to

(a) $2A$ (b) $3A$ (c) $4A$ (d) O

30. If $y = e^{-x}$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $-y$ (b) y (c) x (d) $-x$

31. If $x = t^2 + 1$, $y = 2at$, then $\frac{d^2y}{dx^2}$ at $t = a$ is

(a) $-\frac{1}{a}$ (b) $-\frac{1}{2a^2}$ (c) $\frac{1}{2a^2}$ (d) 0

32. The function $f(x) = \begin{cases} x^2 & \text{for } x < 1 \\ 2-x & \text{for } x \geq 1 \end{cases}$ is

(a) not differentiable at $x = 1$
(b) differentiable at $x = 1$
(c) not continuous at $x = 1$
(d) neither continuous nor differentiable at $x = 1$

33. The curve $x^2 - xy + y^2 = 27$ has tangents parallel to x-axis at

(a) $(3, 6)$ and $(-3, -6)$ (b) $(3, -6)$ and $(-3, 6)$
(c) $(-3, -6)$ and $(3, -6)$ (d) $(-3, 6)$ and $(-3, -6)$

34. A wire of length 20 cm is bent in the form of a sector of a circle. The maximum area that can be enclosed by the wire is

(a) 20 sq.cm (b) 25 sq.cm
(c) 10 sq.cm (d) 30 sq.cm

35. The function $(x - \sin x)$ decreases for

(a) all x (b) $x < \frac{\pi}{2}$
(c) $0 < x < \frac{\pi}{4}$ (d) no value of x

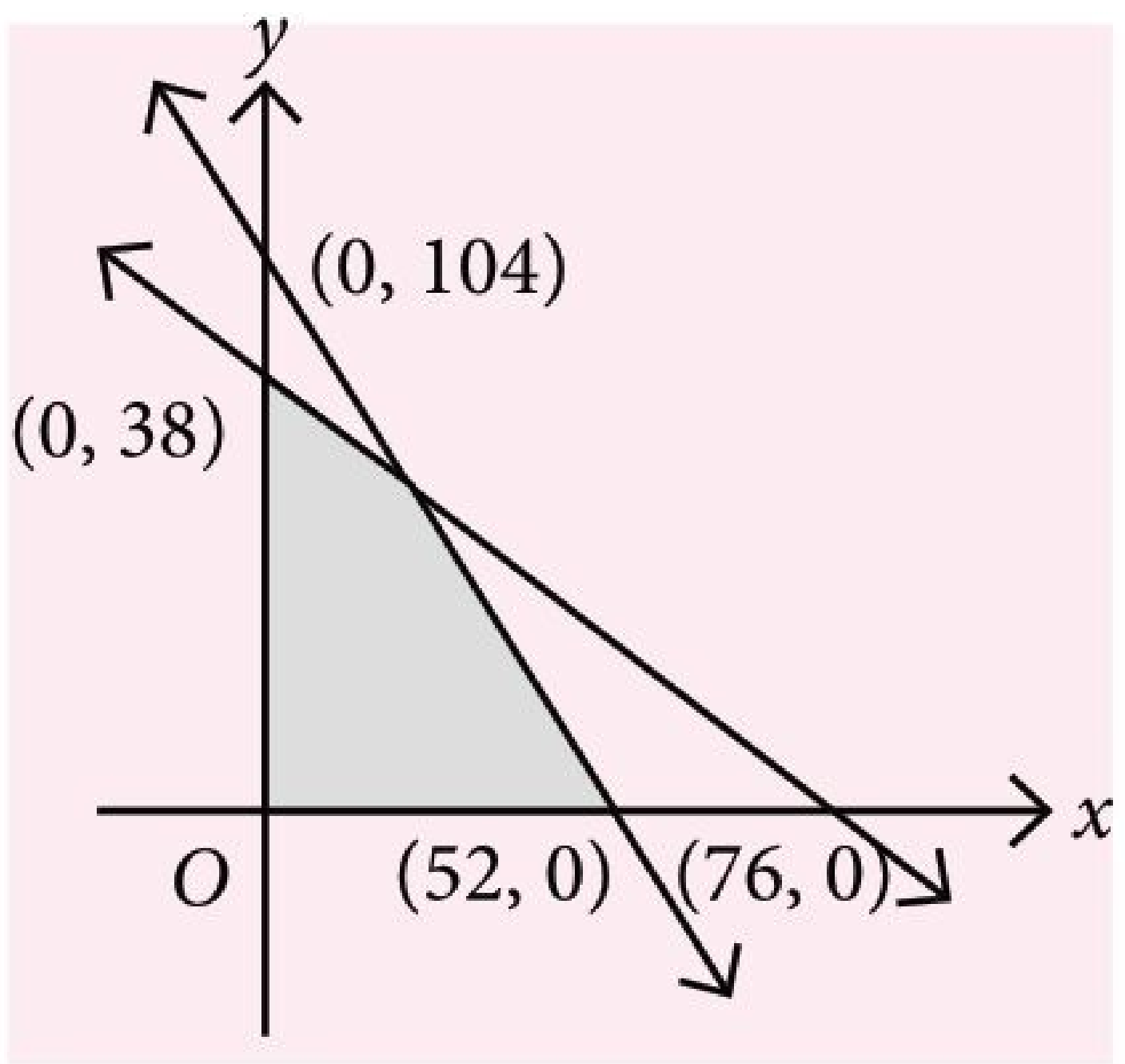
36. If θ is the angle of intersection between the curves $y^2 = 4ax$ and $ay = 2x^2$ at $(a, 2a)$, then the value of $\tan\theta$ is

(a) 3/5 (b) 2/3
(c) 3/4 (d) 2/5

37. The maximum value of $Z = 3x + 4y$ subject to the constraints $x \geq 0$, $y \geq 0$ and $x + y \leq 1$ is

(a) 7 (b) 4 (c) 3 (d) 10

38. The feasible region of an LPP is given in the following figure



Then, the constraints of the LPP are $x \geq 0$, $y \geq 0$ and

(a) $2x + y \leq 52$ and $x + 2y \leq 76$
(b) $2x + y \leq 104$ and $x + 2y \leq 76$
(c) $x + 2y \leq 104$ and $2x + y \leq 76$
(d) $x + 2y \leq 104$ and $2x + y \leq 38$

39. If the minimum value of an objective function $Z = ax + by$ occurs at two points (3, 4) and (4, 3) then

(a) $a + b = 0$ (b) $a = b$
(c) $3a = b$ (d) $a = 3b$

40. For the following LPP
Maximise $Z = 3x + 4y$
Subject to constraints
$x - y \geq -1$, $x \leq 3$, $x \geq 0$, $y \geq 0$
the maximum value is

(a) 0 (b) 4 (c) 25 (d) 30

SECTION-C

In this section, attempt any 8 questions out of Q. No. 41 to 50.

41. A relation R is defined on Z as aRb if and only if $a^2 - 7ab + 6b^2 = 0$. Then, R is

(a) reflexive and symmetric
(b) symmetric but not reflexive
(c) transitive but not reflexive
(d) reflexive but not symmetric

42. The value of $\begin{vmatrix} \underline{\lvert 1} & \underline{\lvert 2} & \underline{\lvert 3} \\ 2\underline{\lvert 2} & 3\underline{\lvert 3} & 4\underline{\lvert 4} \\ \underline{\lvert 3} & \underline{\lvert 4} & \underline{\lvert 5} \end{vmatrix}$ is

(a) 12 (b) -12 (c) 24 (d) -24

43. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then

(a) $a = 1 = b$ (b) $a = \cos 2\theta$, $b = \sin 2\theta$
(c) $a = \sin 2\theta$, $b = \cos\theta$ (d) $a = \cos\theta$, $b = \sin\theta$

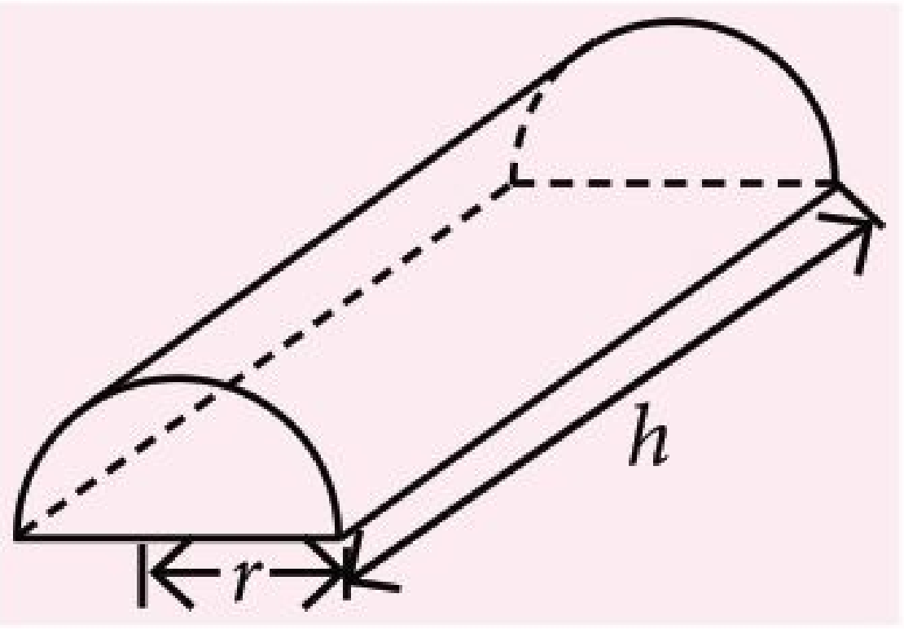
44. The normal to the curve $3y = 6x - 5x^3$ at the point $\left(1, \frac{1}{3}\right)$ passes through the point

(a) (3, 1) (b) (3, 2) (c) (2, 3) (d) (1, 1)

45. If $y = \sin(2\sin^{-1}x)$, then $(1 - x^2)\, y_2$ is equal to

(a) $-xy_1 + 4y$ (b) $-xy_1 - 4y$
(c) $xy_1 - 4y$ (d) $xy_1 + 4y$

CASE STUDY



Some young entrepreneur started as industry "young achievers" for casting metal into various shapes. They put up an advertisement online stating the same and expecting order to cast metal for toys, sculptures, decorative pieces and more.

A group of friends wanted to make innovative toys and hence contacted the "young achievers to order them to cast metal into solid half cylinders with a rectangular base and semi-circular ends.

Based on the above information, answer the following questions :

46. The volume (V) of the casted half cylinder will be

(a) $\pi r^2 h$ (b) $\frac{1}{3}\pi r^2 h$
(c) $\frac{1}{2}\pi r^2 h$ (d) $\pi r^2(r + h)$

47. The total surface area (S) of the casted half cylinder will be

(a) $\pi rh + 2\pi r^2 + rh$ (b) $\pi rh + \pi r^2 + 2rh$
(c) $2\pi rh + \pi r^2 + 2rh$ (d) $\pi rh + \pi r^2 + rh$

48. The total surface area S can be expressed in terms of V and r as

(a) $2\pi r + \frac{2V(\pi+2)}{\pi r}$ (b) $\pi r + \frac{2V}{\pi r}$

(c) $\pi r^2 + \frac{2V(\pi+2)}{\pi r}$ (d) $2\pi r^2 + \frac{2V(\pi+2)}{\pi r}$

49. For the given half-cylinder of volume V, the total surface area S is minimum, when

(a) $(\pi + 2)\, V = \pi^2 r^3$ (b) $(\pi + 2)\, V = \pi^2 r^2$
(c) $2(\pi + 2)\, V = \pi^2 r^3$ (d) $(\pi + 2)\, V = \pi^2 r$

50. The ratio $h : 2r$ for which S to be minimum will be equal to

(a) $2\pi : \pi + 2$ (b) $2\pi : \pi + 1$
(c) $\pi : \pi + 1$ (d) $\pi : \pi + 2$

SOLUTIONS

1. **(c) :** Consider, $R\ \{(x, y) : xy$ is the square number, $x, y, \in N\}$
As, $xx = x^2$, which is the square of an integer x.
$\Rightarrow\ (x, x) \in R$
So, R is reflexive.

2. **(d) :** We have, $f(x) = 4 + 3\cos x,\ \forall x \in R$

At $x = \frac{\pi}{2},\ f\left(\frac{\pi}{2}\right) = 4 + 3\cos\frac{\pi}{2} = 4$

$f\left(-\frac{\pi}{2}\right) = 4 + 3\cos\left(-\frac{\pi}{2}\right) = 4$

Since, $f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$, But $\frac{\pi}{2} \neq -\frac{\pi}{2}$

Therefore, f is not one-one.
As $-1 \le \cos x \le 1,\ \forall x \in R$
$\Rightarrow\ 1 \le 4 + 3\cos x \le 7,\ \forall x \in R$
$\Rightarrow\ f(x) \in [1, 7]$, where $[1, 7]$ is subset of R.
$\therefore\ f$ is not onto.

3. **(b) :** Given, $y = \cot^{-1}x,\ x < 0$
As, $-\infty < x < 0$
$\Rightarrow\ \cot^{-1}\infty > \cot^{-1}x > \cot^{-1}0$
$\Rightarrow\ \pi > \cot^{-1}x > \frac{\pi}{2} \Rightarrow \frac{\pi}{2} < y < \pi$

4. **(d) :** $\because f : X \to Y$ is one-one, if different element of X have different image in Y under f. But here, no such situation is possible.

5. **(d)**

6. **(b) :** We have, $P' = 2P + I$ ...(i)
Now, $(P')' = (2P + I)' = 2P' + I$
$\Rightarrow\ P = 2(2P + I) + I$ (Using (i)]
$\Rightarrow\ P = 4P + 3I \Rightarrow P = -I$

7. **(a) :** Consider $C(A+B')$ *i.e.*, $C_{3\times3}\,(A_{2\times3} + B'_{2\times3}) = C_{3\times3}\,(A + B')_{2\times3}$
Here, number of columns in matric C is 3 and number of rows in matrix $(A + B')$ is 2. So, it is not defined .

8. **(d) :** Given, $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ ...(i)
$|A^3| = 27$
$\Rightarrow\ |A|^3 = 27\ [\because |A^n| = |A|^n] \Rightarrow A = 3$ ...(ii)

From (i) and (ii), we get

$\Rightarrow \quad \alpha^2 - 4 = 3 \;\Rightarrow\; \alpha^2 = 7 \;\Rightarrow\; \alpha = \pm\sqrt{7}$

9. (d) : We have, $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix} = 0$

$\Rightarrow \quad 5(-2x + 18) -3\,(14 + 27) - 1\,(-42 - 9x) = 0$

$\Rightarrow -x + 9 = 0 \;\Rightarrow\; x = 9$

10. (b) : Given, $A = \begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}$

$\therefore \quad |A| = 20 - 21 = -1$

And adj $A = \begin{bmatrix} -5 & -7 \\ -3 & -4 \end{bmatrix}^T = \begin{bmatrix} -5 & -3 \\ -7 & -4 \end{bmatrix}$

$\therefore\ A^{-1} = \dfrac{1}{|A|}(\text{adj}\,A) = \begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$

11. (a) : Given, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 59 & 69 & -1 \end{bmatrix}$

Here, $|A| = -1$

And adj$A = \begin{bmatrix} -1 & 0 & -59 \\ 0 & -1 & -69 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -59 & -69 & 1 \end{bmatrix}$

$$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{adj}A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 59 & 69 & -1 \end{bmatrix} = A$$

12. (b) : Since $f(x)$ is continuous at $x = 5$,

$$\Rightarrow \lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x) = f(5)$$

$\Rightarrow \quad 3(5) - 8 = 2k \;\Rightarrow\; 7 = 2k \;\Rightarrow\; k = \dfrac{7}{2}$

13. (c) : Since, greatest integer function *i.e.*, $[x]$ is continuous at all points except at integers.

$\therefore \quad f(x)$ is continous at 1.5.

14. (a) : Given, $y = \tan^{-1}(e^{2x})$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{1+e^{4x}} \times 2e^{2x} = \frac{2e^{2x}}{1+e^{4x}}$$

15. (a) : Given, $y^2\,(2 - x) = x^3$

$$\Rightarrow \quad y^2 = \frac{x^3}{2-x} \quad \Rightarrow \quad 2y\cdot\frac{dy}{dx} = \frac{(2-x)\times 3x^2 - x^3(-1)}{(2-x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{6x^2 - 2x^3}{2y(2-x)^2} \quad \Rightarrow \left(\frac{dy}{dx}\right)_{(1,\,1)} = \frac{6-2}{2\times 1} = 2$$

16. (a) : Given, $y = x^2 - 5x + 6$

$\Rightarrow \dfrac{dy}{dx} = 2x - 5$

Now, $\left(\dfrac{dy}{dx}\right)_{\text{at }(2,\,0)} = -1 = m_1$

Also, $\left(\dfrac{dy}{dx}\right)_{\text{at }(3,\,0)} = 1 = m_2$

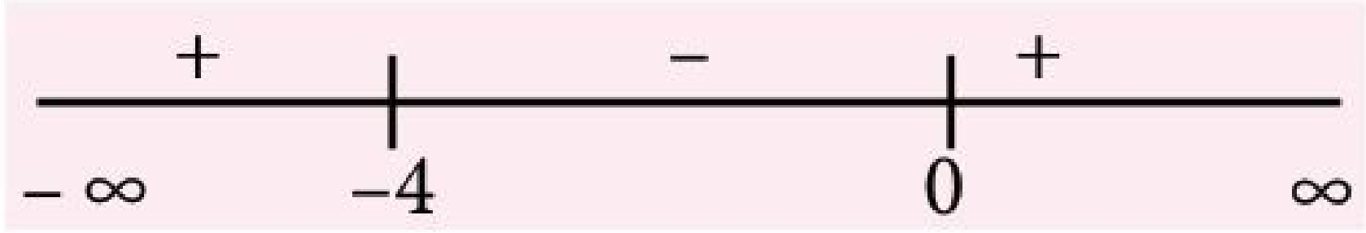
Since, $m_1 m_2 = -1$

So, the slopes are perpendicular

$\therefore \quad \theta = \dfrac{\pi}{2}$

17. (a) : Given, $y = x^3 + 6x^2 + 6 \Rightarrow \dfrac{dy}{dx} = 3x^2 + 12x$

For increasing, $\dfrac{dy}{dx} > 0 \Rightarrow 3x^2 + 12x > 0 \Rightarrow 3x\,(x + 4) > 0$



So, y is strictly increasing in $(-\infty, -4) \cup (0, \infty)$.

18. (b) : Let $f(x) = x - x^2$

$\therefore \quad f'(x) = 1 - 2x$

For critical point, let $f(x) = 0$

$\Rightarrow \quad 1 - 2x = 0 \Rightarrow x = 1/2$

Now, at $x = 1/2$, $f''(x) = -2 < 0$

So, $f(x)$ has maximum value at $x = 1/2$.

19. (a) : Given, $Z = 11x + 7y$

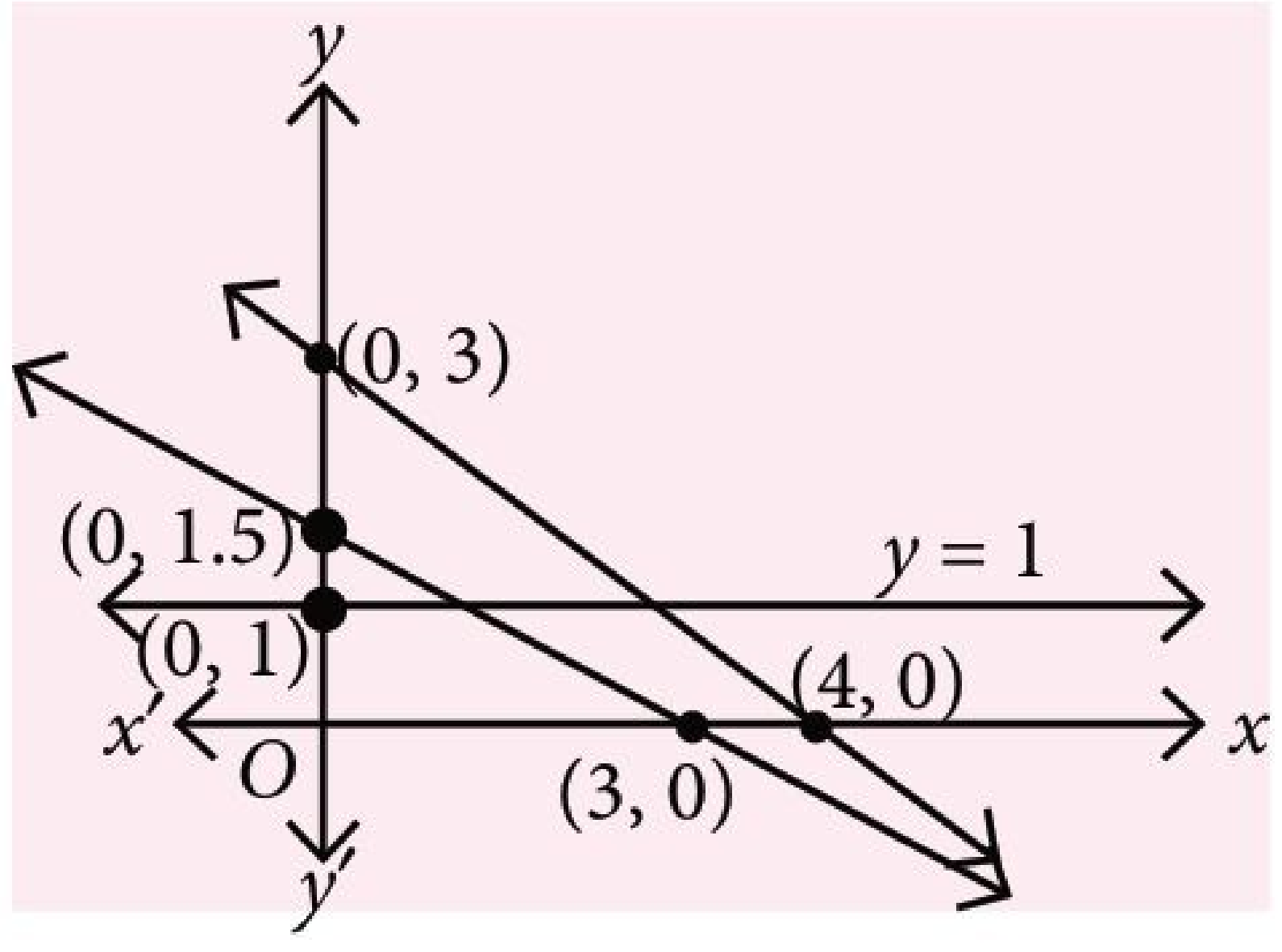
At (0, 3), $Z = 11 \times 0 + 7 \times 3 = 21$

At (3, 2), $Z = 11 \times 3 + 7 \times 2 = 47$

At (0, 5), $Z = 11 \times 0 + 7 \times 5 = 35$

Thus, Z is minimum at (0, 3) and minimum value of Z is 21.

20. (a) : Given,
$x + 2y \le 3$, $3x + 4y \ge 12$, $x \ge 0$, $y \ge 1$
The graph of given constraints is shown here.



Since, there is no common region, so, no solution exists .

21. (c) : Equivalence relations on the set {1, 2, 3} containing the elements (1, 2) and (2, 1) are

R_1 = {(1, 2), (2, 1), (1, 1), (2, 2), (3, 3)}

R_2 = {(1, 2), (2, 1), (1, 3), (3, 1) (2, 3), (3, 2), (1, 1), (2, 2), (3, 3)}

$\therefore \quad$ Number of equivalence relations is 2.

22. (d) : Given $f(x) = \dfrac{1}{x}$, for all $x \in R$

At $x = 0 \in R$, $f(x)$ is undefined.

23. (c) : Given, $f(x) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

Now, $f(1) = \dfrac{1+1}{2} = 1,\ f(2) = \dfrac{2}{2} = 1$

$\Rightarrow$ $f(1) = f(2)$ but $1 \neq 2$ $\Rightarrow$ f is not one-one
But f is onto because range of f is N.

24. (b) : We have,

$$\sin^{-1}\left(\cos\frac{13\pi}{5}\right) = \sin^{-1}\left[\cos\left(2\pi + \frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\cos\frac{3\pi}{5}\right] = \sin^{-1}\left[\cos\left(\frac{\pi}{2} + \frac{\pi}{10}\right)\right]$$

$$= \sin^{-1}\left(-\sin\frac{\pi}{10}\right) = -\sin^{-1}\left(\sin\frac{\pi}{10}\right) = -\frac{\pi}{10}$$

25. (c) : Given $\sin^{-1}x > \cos^{-1}x$

$$\Rightarrow \sin^{-1}x > \frac{\pi}{2} - \sin^{-1}x \quad \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

$$\Rightarrow 2\sin^{-1}x > \frac{\pi}{2} \Rightarrow \sin^{-1}x > \frac{\pi}{4} \Rightarrow x > \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} < x \le 1 \quad \left[\because \sin^{-1}x \le \frac{\pi}{2}\right]$$

26. (b) : We have, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

and $A + A' = I$

$$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 2\cos\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$$

27. (d) : Let $D = \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$

$= (y + k)((y + k)^2 - y^2) - y(y^2 + ky - y^2) + y(y^2 - y^2 - yk)$
$= k^3 + 3k^2y = k^2(k + 3y)$

28. (d) : $|A| = \begin{vmatrix} 1 & -2 & 4 \\ 2 & -1 & 3 \\ 4 & 2 & 0 \end{vmatrix}$

$= 1(0 - 6) + 2(0 - 12) + 4(4 + 4) = 2$ ...(i)
Given, $A = \text{adj}B$
$\Rightarrow$ $|A| = |\text{adj}B|$ $\Rightarrow$ $|\text{adj}B| = 2$ (Using (i))

$\Rightarrow$ $|B|^2 = 2$
[$\because$ $|\text{adj}B| = |B|^{3-1}$, where B is 3×3 matrix]
$\Rightarrow$ $|B| = \pm\sqrt{2}$

$$\therefore\ B^{-1} = \pm\frac{1}{\sqrt{2}}A \quad \left[\because B^{-1} = \frac{1}{|B|}(\text{adj}B)\right]$$

29. (d)

30. (b) : Given, $y = e^{-x}$

$$\Rightarrow \frac{dy}{dx} = -e^{-x} \Rightarrow \frac{d^2y}{dx^2} = e^{-x} = y$$

31. (b) : Given, $x = t^2 + 1$ and $y = 2at$

$$\Rightarrow \frac{dx}{dt} = 2t \Rightarrow \frac{dy}{dt} = 2a \quad \therefore \frac{dy}{dx} = \frac{a}{t}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-a}{t^2}\cdot\frac{dt}{dx} = \frac{-a}{2t^3} \quad \therefore \left(\frac{d^2y}{dx^2}\right)_{\text{at } t = a} = \frac{-a}{2a^3} = \frac{-1}{2a^2}$$

32. (a) : At $x = 1$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} x^2 = 1$$

And $\displaystyle\lim_{x\to 1^+} f(x) = \lim_{x\to 1} 2 - x = 1$

Also, $f(1) = 2 - 1 = 1$

$$\because \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(x)$$

$\therefore$ $f(x)$ is continuous at $x = 1$

Now, $\text{L.H.D} = \displaystyle\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1}$

$$= \lim_{x\to 1} \frac{x^2 - 1}{x - 1} = \lim_{x\to 1} (x + 1) = 2$$

$$\text{R.H.D} = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1} \frac{(2 - x) - 1}{x - 1} = -1$$

$\because$ L.H.D ≠ R.H.D

$\therefore$ $f(x)$ is not differentiable at $x = 1$

33. (a) : Given, $x^2 - xy + y^2 = 27$ ...(i)
Differentiating (i) w.r.t. x, we get

$$2x - x\frac{dy}{dx} - y + 2y\cdot\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

Since, curve has tangent parallel to x-axis

$$\therefore \frac{dy}{dx} = 0 \Rightarrow \frac{2x - y}{x - 2y} = 0 \Rightarrow 2x - y = 0 \Rightarrow y = 2x$$

Substituting, $y = 2x$ in (i), we get

$x^2 - x\cdot(2x) + (2x)^2 = 27 \Rightarrow 3x^2 = 27 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

Hence, the required points are (3, 6) and (–3, –6).

34. (b) : Let r be the radius, θ be the central angle and l be the length of the circular sector.

Given, $l + 2r = 20$

$$\Rightarrow \quad r\theta + 2r = 20 \quad (\because \; l = r\theta) \;\Rightarrow\; \theta = \frac{20-2r}{r}$$

Let A be the area of the circular sector.

$$\therefore \quad A = \frac{\pi r^2\theta}{2\pi} = \frac{r^2}{2}\cdot\left(\frac{20-2r}{r}\right) = r(10-r)$$

$$\Rightarrow \quad \frac{dA}{dr} = 10-2r$$

For maximum or minimum value of A, we have

$$\frac{dA}{dr} = 0 \Rightarrow r = 5 \text{ and } \frac{d^2A}{dr^2} = -2 < 0$$

$\therefore$ Area is maximum at $r = 5$

$\therefore$ Maximum area, $A = 5\,(10 - 5) = 25\text{ cm}^2$

35. (d) : Let $f(x) = x - \sin x$

Differentiating w.r.t. x, we get $f'(x) = 1 - \cos x$

For function to be decreasing, $f'(x) < 0$

$\Rightarrow \; 1 - \cos x < 0 \;\Rightarrow\; \cos x > 1$,

which is not possible, because maximum value of $\cos x$ is 1.

$\therefore \; f(x) = (x - \sin x)$ doesn't decrease at any value of x.

36. (a) : Given curves are $y^2 = 4ax$ ... (i)

and $ay = 2x^2$ ... (ii)

Now, differentiating (i) w.r.t. x, we get

$$2y\frac{dy}{dx} = 4a \Rightarrow \left(\frac{dy}{dx}\right)_{(a,\,2a)} = \frac{2a}{2a} = 1 = m_1 \text{ (say)}$$

Again, differentiating (ii) w.r.t. x, we get

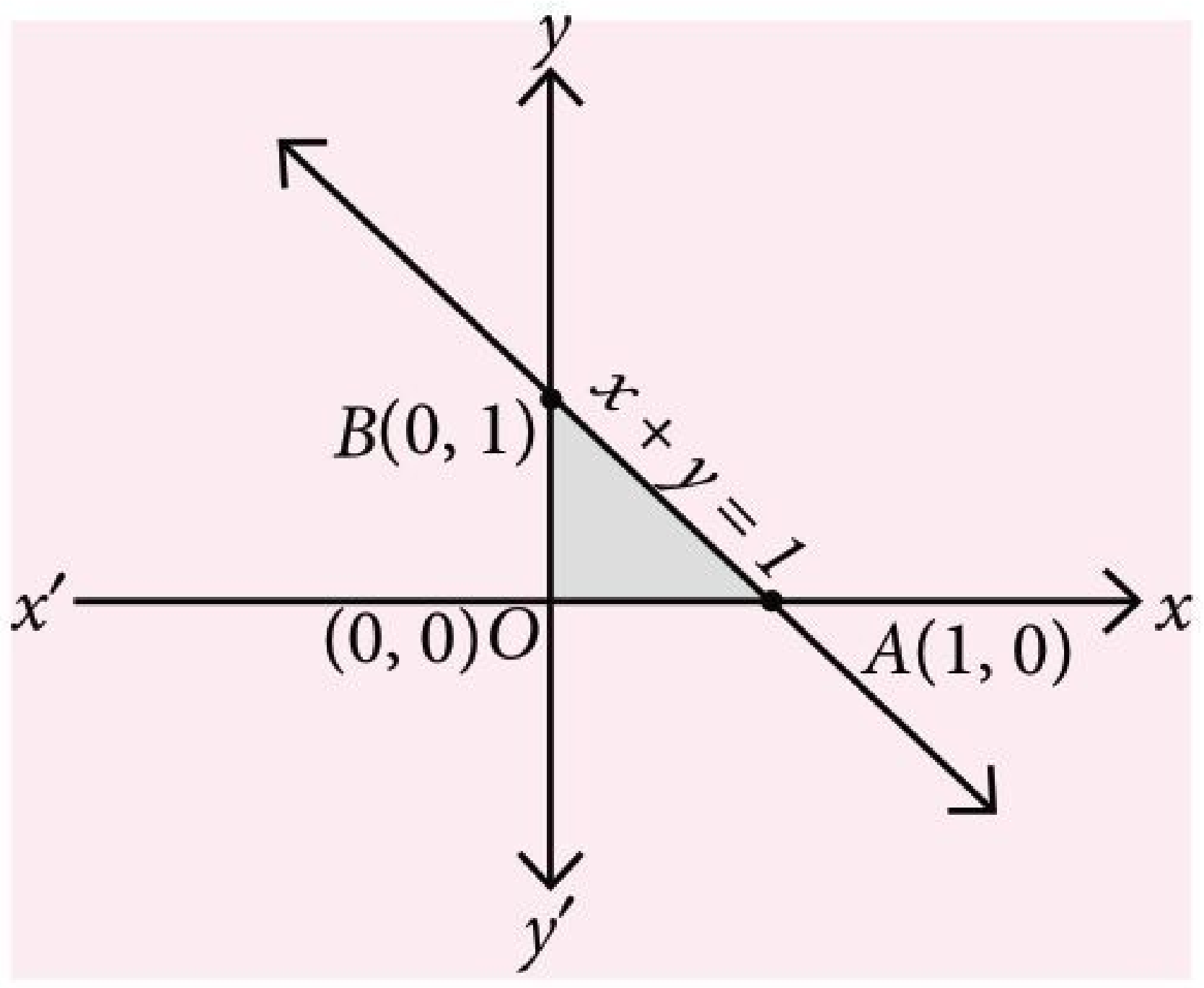
$$a\frac{dy}{dx} = 4x \Rightarrow \left(\frac{dy}{dx}\right)_{(a,\,2a)} = \frac{4a}{a} = 4 = m_2 \text{ (say)}$$

$$\therefore \tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right| = \left|\frac{4-1}{1+4}\right| \Rightarrow \tan\theta = \frac{3}{5}$$

37. (b) : We have to maximise $Z = 3x + 4y$

Subject to constraints,

$x \geq 0,\; y \geq 0$ and $x + y \leq 1$



The shaded portion OAB is the feasible region, where $O(0, 0)$, $A(1, 0)$ and $B(0, 1)$ are the corner points.

At $O(0, 0)$, $Z = 3 \times 0 + 4 \times 0 = 0$

At $A(1, 0)$, $Z = 3 \times 1 + 4 \times 0 = 3$

At $B(0, 1)$, $Z = 3 \times 0 + 4 \times 1 = 4$

$\therefore$ Maximum value of Z is 4, which occurs at $B(0, 1)$.

38. (b) : Clearly, the pair of points given in graph, and (0, 104); (52, 0) and (0, 38); (76, 0) satisfy the corresponding equations given in option(b) *i.e.*, $2x + y \leq 104$ and $x + 2y \leq 76$.

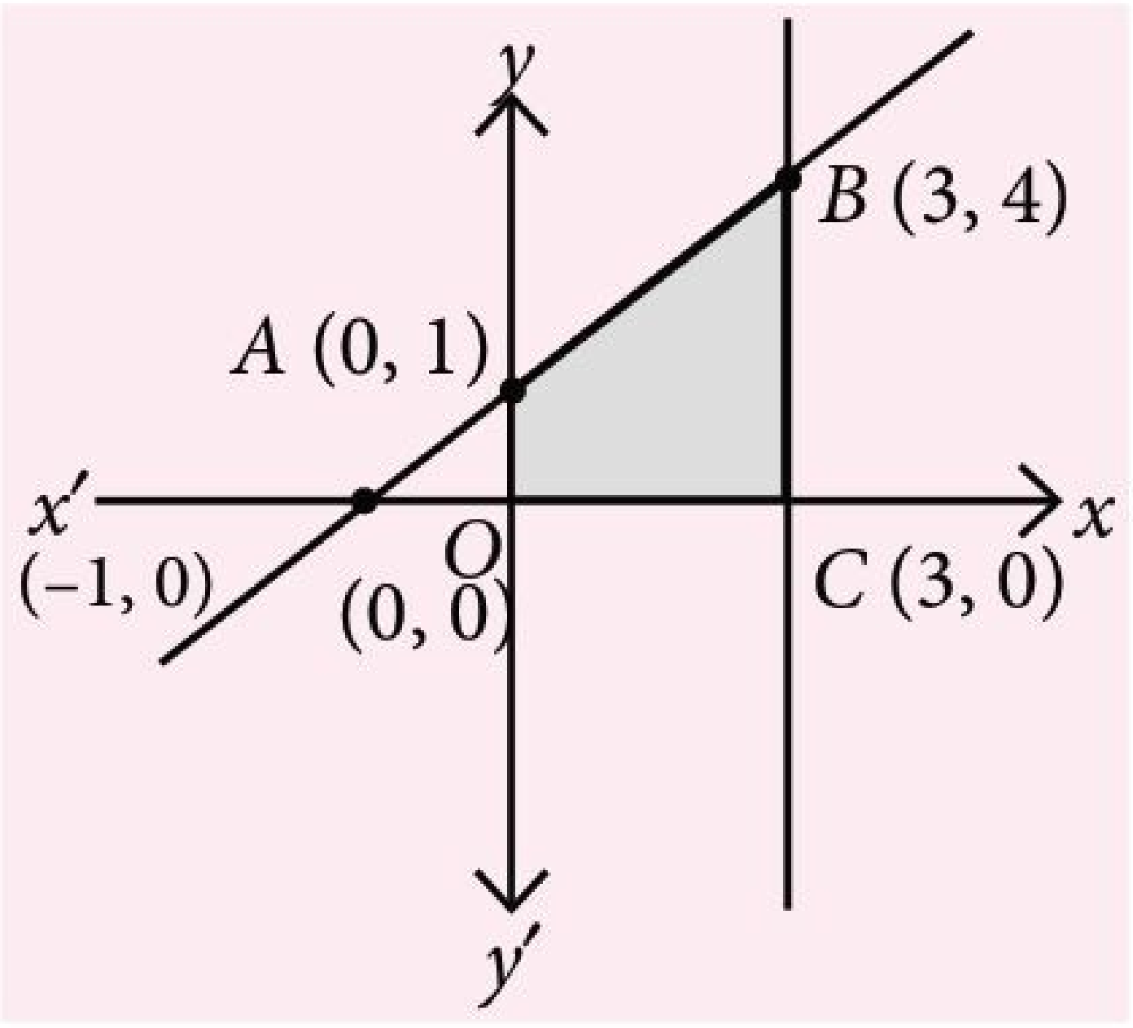
39. (b) : Since, minimum value of $Z = ax + by$ occurs at two points (3, 4) and (4, 3).

$\therefore \quad 3a + 4b = 4a + 3b \;\Rightarrow\; a = b$

40. (c) : Given, $Z = 3x + 4y$

Subject to constraints

$x - y \geq -1,\; x \leq 3;\; x \geq 0,\; y \geq 0$



The shaded region $OABC$ is the feasible region, where corner points are $O(0, 0)$, $A(0, 1)$, $B(3, 4)$ and $C(3, 0)$

At O (0, 0), $Z = 3(0) + 4\,(0) = 0$

At A (0, 1), $Z = 3(0) + 4\,(1) = 4$

At B (3, 4), $Z = 3(3) + 4\,(4) = 25$

At C (3, 0), $Z = 3(3) + 4\,(0) = 9$

$\therefore$ Maximum value of Z is 25, which occurs at $B(3, 4)$.

41. (d) : Given, aRb, $a\ b \in Z$

Reflexive : For $a \in Z$, we have

$a^2 - 7a\cdot a + 6a^2 = A^2 - 7a^2 + 6a^2 = 0 \Rightarrow (a, a) \in R$

$\therefore$ Relation is reflexive.

Symmetric : Since, $(6, 1) \in R$

As $6^2 - 7 \times 6 \times 1 + 6 \times 1^2 = 36 - 42 + 6 = 0$

But $(1, 6) \notin R$

$\therefore$ Relation is not symmetric.

42. (d) : Given, $\begin{vmatrix} \underline{|1} & \underline{|2} & \underline{|3} \\ 2\underline{|2} & 3\underline{|3} & 4\underline{|4} \\ \underline{|3} & \underline{|4} & \underline{|5} \end{vmatrix} = \begin{vmatrix} 1 & 2 & 6 \\ 4 & 18 & 96 \\ 6 & 24 & 120 \end{vmatrix}$

$= 1(2160 - 2304) - 2(480 - 576) + 6(96 - 108)$

$= -144 - 2\,(-96) + 6\,(-12) = -144 + 192 - 72 = -24$

43. (b) : We have,

$$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

Now, $\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \dfrac{1}{\sec^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta & -\tan\theta\cos^2\theta \\ \cos^2\theta\,\tan\theta & \cos^2\theta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} \cos^2\theta & -\tan\theta\cos^2\theta \\ \cos^2\theta\tan\theta & \cos^2\theta \end{bmatrix}$$

$$=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos^2\theta-\cos^2\theta\tan^2\theta & -2\tan\theta\cos^2\theta \\ 2\tan\theta\cos^2\theta & \cos^2\theta-\cos^2\theta\tan^2\theta \end{bmatrix}$$

$$=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$\therefore\quad a = \cos^2\theta - \cos^2\theta\tan^2\theta$ and $b = 2\tan\theta\cos^2\theta$

$$\Rightarrow\quad a=\cos^2\theta\left(1-\frac{\sin^2\theta}{\cos^2\theta}\right) \text{ and } b=\frac{2\sin\theta}{\cos\theta}\cdot\cos^2\theta$$

$\Rightarrow\quad a = \cos^2\theta - \sin^2\theta = \cos 2\theta$ and $b = 2\sin\theta\cos\theta = \sin 2\theta$

44. (a) : Given, $3y = 6x - 5x^3$

$$\Rightarrow\quad y = 2x - \frac{5}{3}x^3 \quad\Rightarrow\quad \frac{dy}{dx}=2-5x^2$$

$$\therefore\quad \left(\frac{dy}{dx}\right)_{\text{at}\left(1,\ \frac{1}{3}\right)}=-3$$

$\therefore\quad$ Equation of normal at $\left(1,\ \frac{1}{3}\right)$ is

$$\left(y-\frac{1}{3}\right)=\frac{1}{3}(x-1)\quad\left[\because\ \frac{dy}{dx}=-3\right]$$

$\Rightarrow 3y - 1 = x - 1$

$\Rightarrow\quad x - 3y = 0$ which passes through point (3 1)

45. (c) : We have, $y = \sin(2\sin^{-1}x)$

$$\Rightarrow\quad y=\sin\left[\sin^{-1}\left(2x\sqrt{1-x^2}\right)\right]$$

$$\left[\because\ 2\sin^{-1}x=\sin^{-1}2x\sqrt{1-x^2}\right]$$

$$\Rightarrow\quad y=2x\sqrt{1-x^2} \qquad \text{...(i)}$$

$$\Rightarrow\quad y_1=2x\times\frac{-2x}{2\sqrt{1-x^2}}+2\sqrt{1-x^2}=\frac{-4x^2+2}{\sqrt{1-x^2}} \qquad \text{...(ii)}$$

$$\therefore\quad y_2=\frac{\sqrt{1-x^2}(-8x)-(-4x^2+2)\times\dfrac{-2x}{2\sqrt{1-x^2}}}{1-x^2}$$

$$=\frac{4x^3-6x}{(1-x^2)\sqrt{1-x^2}}$$

$$\Rightarrow\quad (1-x^2)y_2=\frac{4x^3-6x}{\sqrt{1-x^2}}$$

Now, consider $xy_1 - 4y$

$$=\frac{-4x^2+2x}{\sqrt{1-x^2}}-8x\sqrt{1-x^2} \qquad \text{[Using (i) and (ii)]}$$

$$=\frac{4x^3-6x}{\sqrt{1-x^2}}$$

Thus, $(1 - x^2)\, y_2 = xy_1 - 4y$

46. (c) : $\because$ Volume of cylinder $= \pi r^2 h$

$\therefore\quad V =$ Volume of casted half cylinder $= (1/2)\pi r^2 h$

47. (b) : Total surface area, $S = \dfrac{2\pi r(r+h)}{2}+2rh$

$= \pi r^2 + \pi rh + 2rh$

48. (c) : Here, $S = \pi r^2+\dfrac{2V(\pi+2)}{\pi r}$

$$\left[\because\ V=\frac{1}{2}\pi r^2 h\ \Rightarrow\ \frac{2V}{\pi r}=rh\right]$$

49. (a) : $\because\ S=\pi r^2+\dfrac{2V(\pi+2)}{\pi r}$

$$\Rightarrow\quad \frac{dS}{dr}=2\pi r-\frac{2V(\pi+2)}{\pi}\times\frac{1}{r^2}$$

For S to be minimum, $\dfrac{dS}{dr}=0$

$$\Rightarrow\quad 2\pi r=\frac{2V(\pi+2)}{\pi r^2}\quad\Rightarrow\quad \pi^2r^3=V(\pi+2)$$

50. (d): $\because\ V=\dfrac{1}{2}\pi r^2 h$ ...(i)

and S will be minimum, when $(\pi + 2)\, V = \pi^2 r^3$

$$\Rightarrow\quad V=\frac{\pi^2r^3}{\pi+2} \qquad \text{...(ii)}$$

From (i) and (ii), we get

$$\Rightarrow\quad \frac{1}{2}\pi r^2 h=\frac{\pi^2r^3}{\pi+2}\quad\Rightarrow \pi r^2 h\,(\pi+2)=2\pi^2r^3$$

$$\Rightarrow\quad h(\pi+2)=2\pi r\quad\Rightarrow\quad \frac{h}{2r}=\frac{\pi}{\pi+2}$$

Thus, required ratio *i.e.*, $h : 2r$ is $\pi : \pi + 2$.

⁂

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1. (c) **2.** (a) **3.** (b) **4.** (b) **5.** (c)
6. (c) **7.** (a,b,c,d) **8.** (a, c) **9.** (a,c)
10. (a,d) **11.** (a,c) **12.** (a,b,c,d) **13.** (b)
14. (c) **15.** (c) **16.** (c) **17.** (0) **18.** (4)
19. (4) **20.** (6)

CBSE* warm-up!

Term-II

CLASS-XII

Series 1 Integrals | Application of Integrals | Differential Equations

Time allowed : 2 Hours
Maximum marks : 40

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 8 marks and Part-B carries 32 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 4 MCQs.*
3. *Section-II contains 1 case study-based questions.*

Part - B :

1. *It consists of four Sections-III, IV, V and VI.*
2. *Section-III comprises of 5 questions of 1 mark each.*
3. *Section-IV comprises of 4 questions of 2 marks each.*
4. *Section-V comprises of 3 questions of 3 marks each.*
5. *Section-VI comprises of 2 questions of 5 marks each.*
6. *Internal choice is provided in 1 question of Section-III, 1 question of Section-IV, 1 question of Section-V and 2 questions of section-VI. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Evaluate : $\int (e^{x\log a} + e^{a\log x} + e^{a\log a})dx$

(a) $\frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + a^a x + C$

(b) $\frac{a^x}{\log a} + \frac{x^{a+1}}{a-1} + ax^a + C$

(c) $\frac{a^x}{\log a} + \frac{x^a}{a+1} + ax^a + C$

(d) $\frac{a^x}{\log x} + \frac{x^{a+1}}{a+1} + a^a x + C$

2. The area of the ellipse $\frac{x^2}{4^2} + \frac{y^2}{9^2} = 1$ is

(a) 6π sq. units (b) $\frac{\pi(a^2+b^2)}{4}$ sq. units

(c) $\pi(a + b)$ sq. units (d) 36π sq. units

3. The general solution of the differential equation $x(1 + y^2)\,dx + y\,(1 + x^2)\,dy = 0$ is

(a) $(1 + x^2)(1 + y^2) = 0$

(b) $(1 + x^2)(1 + y^2) = C$

(c) $(1 + x^2) = C(1 + y^2)$

(d) $(1 + y^4) = C(1 + x^2)$

***This is for practice purpose. CBSE has yet not released the official sample paper. So, the pattern is suggestive only. For latest information visit www.cbse.gov.in.**

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4. Evaluate : $\int \frac{1}{\sqrt{9+8x-x^2}}dx$

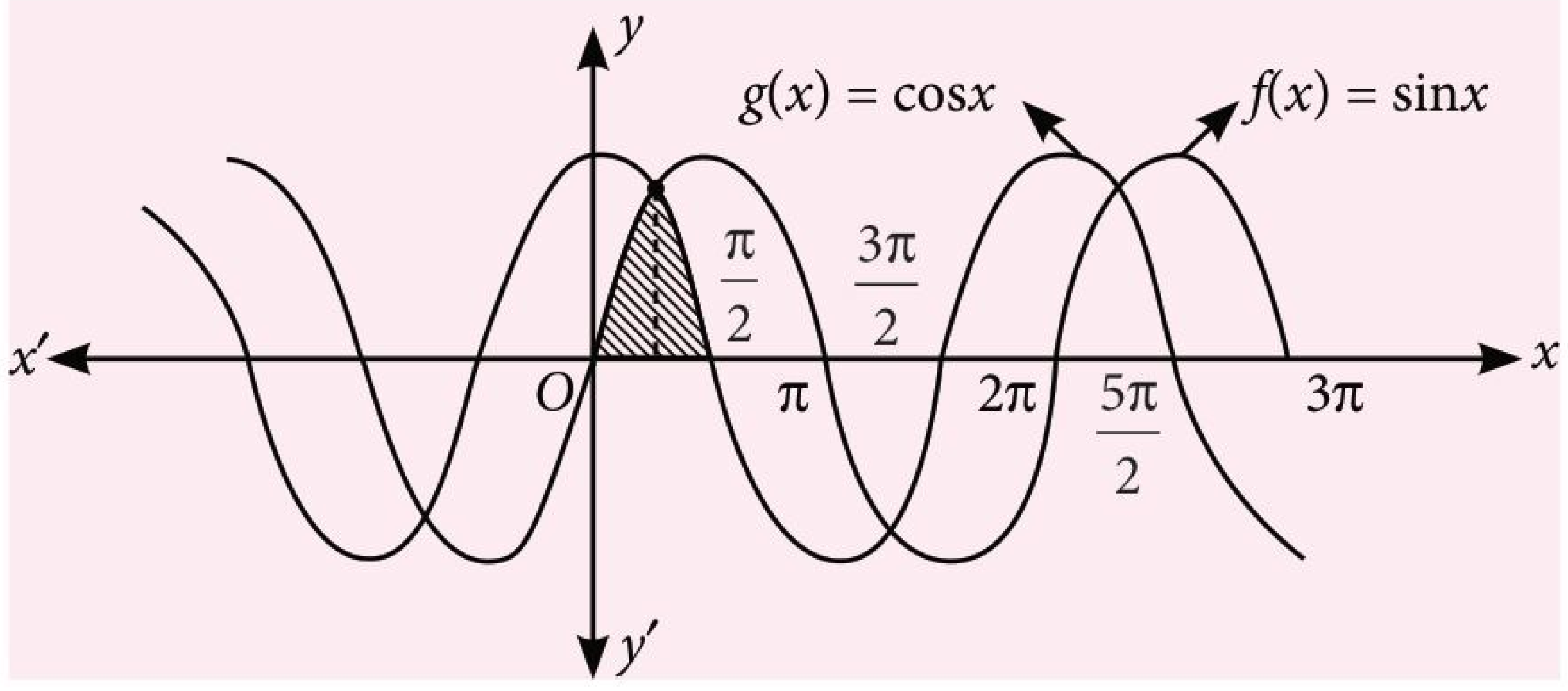
(a) $-\sin^{-1}\left(\frac{x-4}{5}\right)+C$ (b) $\sin^{-1}\left(\frac{x+4}{5}\right)+C$

(c) $\sin^{-1}\left(\frac{x-4}{5}\right)+C$ (d) None of these

Section - II

Case study-based question is compulsory. Attempt any 4 sub parts. Each sub-part carries 1 mark.

5. Graphs of two function $f(x) = \sin x$ and $g(x) = \cos x$ are given below :



Based on the above information, answer the following questions.

(i) In [0, π], the curves $f(x) = \sin x$ and $g(x) = \cos x$ intersect at $x =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) π

(ii) Value of $\int_0^{\pi/4} \sin x\,dx$ is

(a) $1-\frac{1}{\sqrt{2}}$ (b) $1+\frac{1}{\sqrt{2}}$

(c) $2-\frac{1}{\sqrt{2}}$ (d) $2+\frac{1}{\sqrt{2}}$

(iii) Value of $\int_{\pi/4}^{\pi/2} \cos x\,dx$ is

(a) $1+\frac{1}{\sqrt{2}}$ (b) $1-\frac{1}{\sqrt{2}}$

(c) $2-\sqrt{2}$ (d) $2+\sqrt{2}$

(iv) Value of $\int_0^{\pi} \sin x\,dx$ is

(a) 0 (b) 1 (c) 2 (d) –2

(v) Value of $\int_0^{\pi/2} \sin x\,dx$ is

(a) 0 (b) 1 (c) 3 (d) 4

PART - B

Section - III

6. Find the sum of the order and degree of the differential equation $\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\}=0.$

7. Using integration, find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.

8. Find : $\int \frac{2x}{\sqrt[3]{x^2+1}}dx$

OR

Evaluate : $\int_e^{e^2} \frac{dx}{x\log x}$

9. Find the integrating factor of the differential equation $(1+x^2)+(2xy-\cot x)\frac{dx}{dy}=0.$

10. Find the area enclosed between the curve $x^2 + y^2 = 16$ and the coordinate axes in the first quadrant.

Section - IV

11. Evaluate : $\int_1^2\left[\frac{1}{x}-\frac{1}{2x^2}\right]e^{2x}dx$

12. Find the area bounded by the curve $y^2 = 9x$ and the lines $x = 1$, $x = 4$ and $y = 0$ in the first quadrant.

OR

Find area of the region bounded by the curve $y = x^2$ and the line $y = 4$.

13. Solve the following differential equation:

$$\frac{dy}{dx}+y=\cos x-\sin x$$

14. Evaluate : $\int \frac{x^2+x+1}{(x^2+1)(x+2)}dx$

Section - V

15. Find the area of the region bounded by the parabola $y = x^2 + 1$ and the straight line $x + y = 3$.

16. Evaluate : $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}dx$

OR

Find the value of $\int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$.

17. Find the particular solution of the differential equation $(1 - y^2)(1 + \log x)dx + 2xy\,dy = 0$, given that $y = 0$ when $x = 1$.

Section - VI

18. Solve the following differential equation :

$$\frac{dy}{dx}-3y\ \cot x = \sin 2x \text{ given } y = 2 \text{ when } x=\frac{\pi}{2}$$

OR

Solve the following differential equation :

$$\left(x\sin^2\left(\frac{y}{x}\right)-y\right)dx+xdy=0 \quad \text{given} \quad y=\frac{\pi}{4} \text{ when}$$

$x = 1$

19. Evaluate : $\int \frac{3x+5}{x^3-x^2-x+1}dx$

OR

Evaluate : $\int_0^{\pi} \frac{xdx}{a^2\cos^2 x+b^2\sin^2 x}$

SOLUTIONS

1. **(a) :** We have, $\int\left(e^{x\log a}+e^{a\log x}+e^{a\log a}\right)dx$

$$=\int\left(e^{\log a^x}+e^{\log x^a}+e^{\log a^a}\right)dx$$

$$=\int(a^x+x^a+a^a)dx \qquad [\because\ e^{\log\lambda}=\lambda]$$

$$=\int a^x dx+\int x^a dx+\int a^a dx=\frac{a^x}{\log a}+\frac{x^{a+1}}{a+1}+a^a\,x+C$$

2. **(d) :** Since, area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab.

$\therefore$ Required area = $\pi \times 4 \times 9 = 36\pi$ sq. units.

3. **(b) :** Given differential equation is

$x(1 + y^2)dx + y(1 + x^2)dy = 0$

$$\Rightarrow \quad \frac{x}{1+x^2}dx+\frac{y}{1+y^2}dy=0$$

Integrating both sides, we get

$$\frac{1}{2}\log(1+x^2)+\frac{1}{2}\log(1+y^2)=k$$

$\Rightarrow$ $\log(1 + x^2)(1 + y^2) = 2k \Rightarrow (1 + x^2)(1 + y^2) = e^{2k} = C$

4. **(c) :** We have, $\int\frac{1}{\sqrt{9+8x-x^2}}dx$

$$=\int\frac{1}{\sqrt{-\{x^2-8x-9\}}}dx$$

$$=\int\frac{1}{\sqrt{-\{(x-4)^2-5^2\}}}dx=\int\frac{1}{\sqrt{5^2-(x-4)^2}}\,dx$$

$$=\sin^{-1}\left(\frac{x-4}{5}\right)+C$$

5. **(i) (c) :** For point of intersection, we have

$\sin x = \cos x$

$$\Rightarrow \quad \frac{\sin x}{\cos x}=1 \ \Rightarrow\ \tan x=1 \ \Rightarrow\ x=\frac{\pi}{4}$$

(ii) (a) : $\int_0^{\pi/4}\sin x\,dx=[-\cos x]_0^{\pi/4}=-\cos\frac{\pi}{4}+\cos 0$

$$=1-\frac{1}{\sqrt{2}}$$

(iii) (b) : $\int_{\pi/4}^{\pi/2}\cos x\,dx=[\sin x]_{\pi/4}^{\pi/2}=\sin\frac{\pi}{2}-\sin\frac{\pi}{4}$

$$=1-\frac{1}{\sqrt{2}}$$

(iv) (c) : $\int_0^{\pi}\sin x\,dx=[-\cos x]_0^{\pi}$ = [–cos π + cos 0] = 2

(v) (b) : $\int_0^{\pi/2}\sin x\,dx=[-\cos x]_0^{\pi/2}=\left[-\cos\frac{\pi}{2}+\cos 0\right]$

= 0 + 1 = 1

6. The given differential equation is

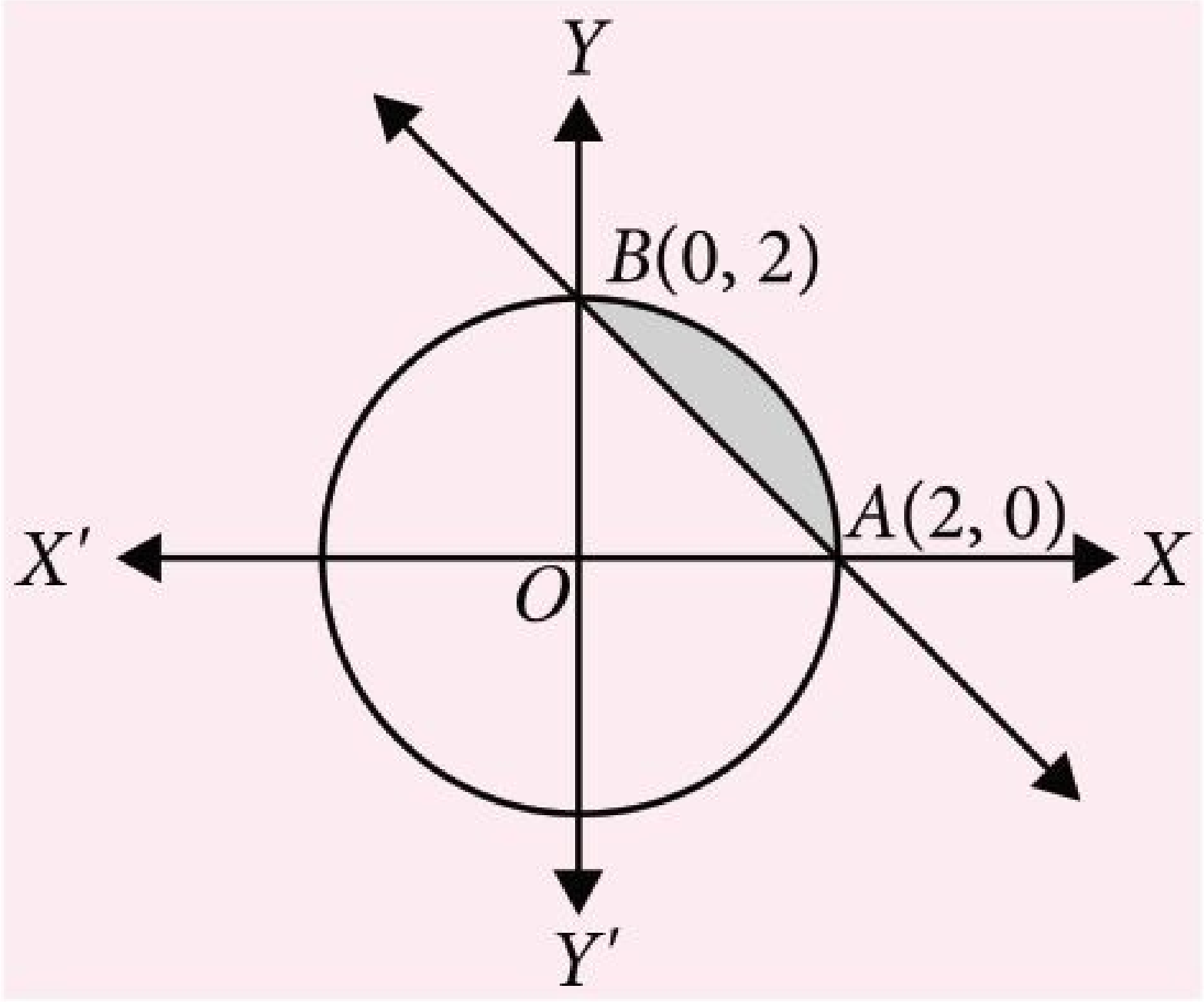
$$\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\}=0 \ \Rightarrow\ 3.\left(\frac{dy}{dx}\right)^2.\frac{d^2y}{dx^2}=0$$

Order = 2 and Degree = 1

$\therefore$ Order + Degree = 2 + 1 = 3

7. The given curves are

$x^2 + y^2 = 4$ ...(i) and $x + y = 2$ ...(ii)



$$\therefore \quad \text{Required area}=\int_0^2\left[\sqrt{4-x^2}-(2-x)\right]dx$$

$$=\left[\frac{x\sqrt{4-x^2}}{2}+\frac{4}{2}\sin^{-1}\frac{x}{2}-2x+\frac{x^2}{2}\right]_0^2$$

$$=0+2\sin^{-1}(1)-4+2-0=2\cdot\frac{\pi}{2}-2=(\pi-2)\text{ sq.units.}$$

8. Let $I=\int\frac{2x}{\sqrt[3]{x^2+1}}dx$

Put $x^2 + 1 = z \Rightarrow 2x\,dx = dz$

$$\therefore \quad I=\int\frac{dz}{(z)^{1/3}}=\int z^{-1/3}dz=\frac{z^{(-1/3)+1}}{(-1/3)+1}+C$$

$$=\frac{3}{2}(x^2+1)^{2/3}+C$$

OR

Let $I=\int_{e}^{e^2}\frac{dx}{x\log x}$

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$

Also $x = e \Rightarrow t = \log e = 1$

and $x = e^2 \Rightarrow t = \log e^2 = 2\log e = 2$

$\therefore \quad I=\int_{1}^{2}\frac{dt}{t}=[\log t]_1^2=\log 2-\log 1=\log 2$

9. The given differential equation is

$$(1+x^2)+(2xy-\cot x)\frac{dx}{dy}=0$$

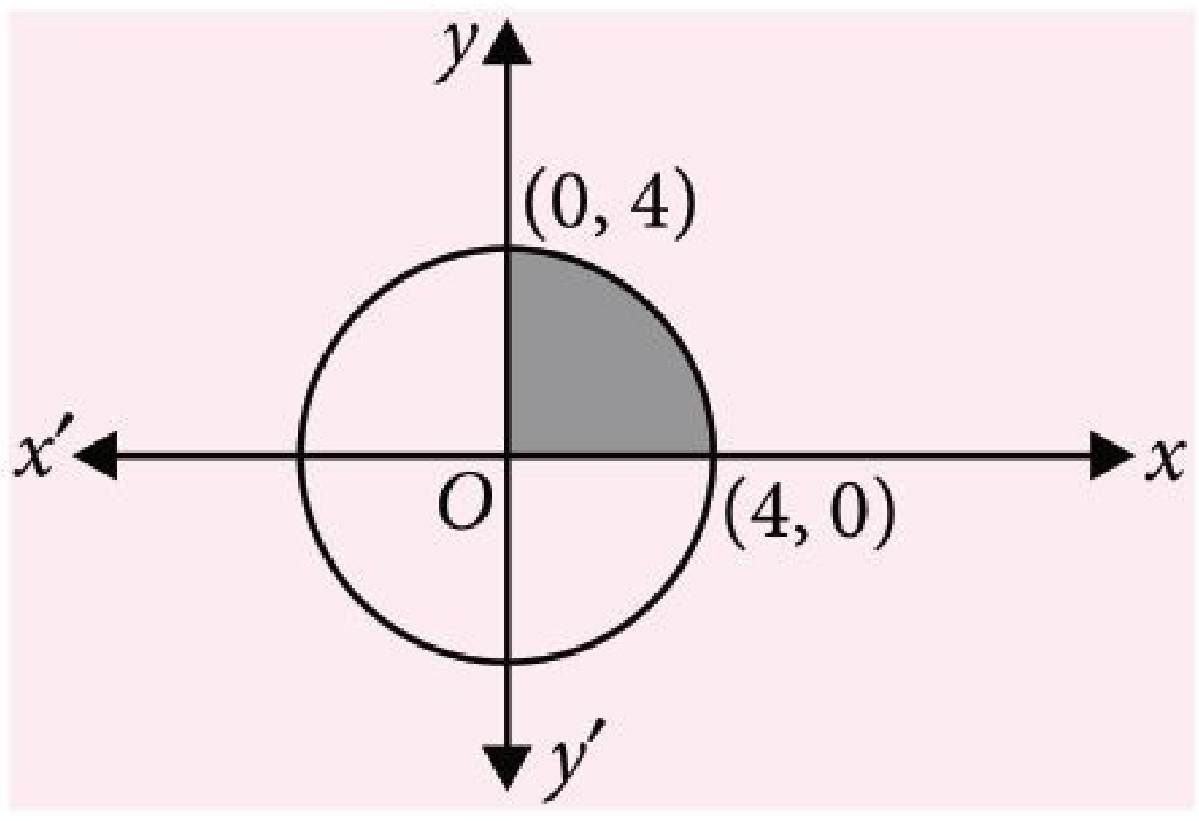
$$\Rightarrow (1+x^2)\frac{dy}{dx}+2xy-\cot x=0$$

$$\Rightarrow \frac{dy}{dx}+\frac{2x}{1+x^2}\cdot y=\frac{\cot x}{1+x^2}$$

This is a linear differential equation of the form $\frac{dy}{dx}+Py=Q$, where, $P=\frac{2x}{1+x^2}$ and $Q=\frac{\cot x}{1+x^2}$

$\therefore$ I.F. $=e^{\int P\,dx}=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$

10. Given curve is a circle with centre (0, 0) and radius 4.



$\therefore$ Required area

$$=\int_0^4\sqrt{16-x^2}\,dx$$

$$=\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4=4\pi \text{ sq. units}$$

11. Let $I=\int_1^2\left[\frac{1}{x}-\frac{1}{2x^2}\right]e^{2x}dx$

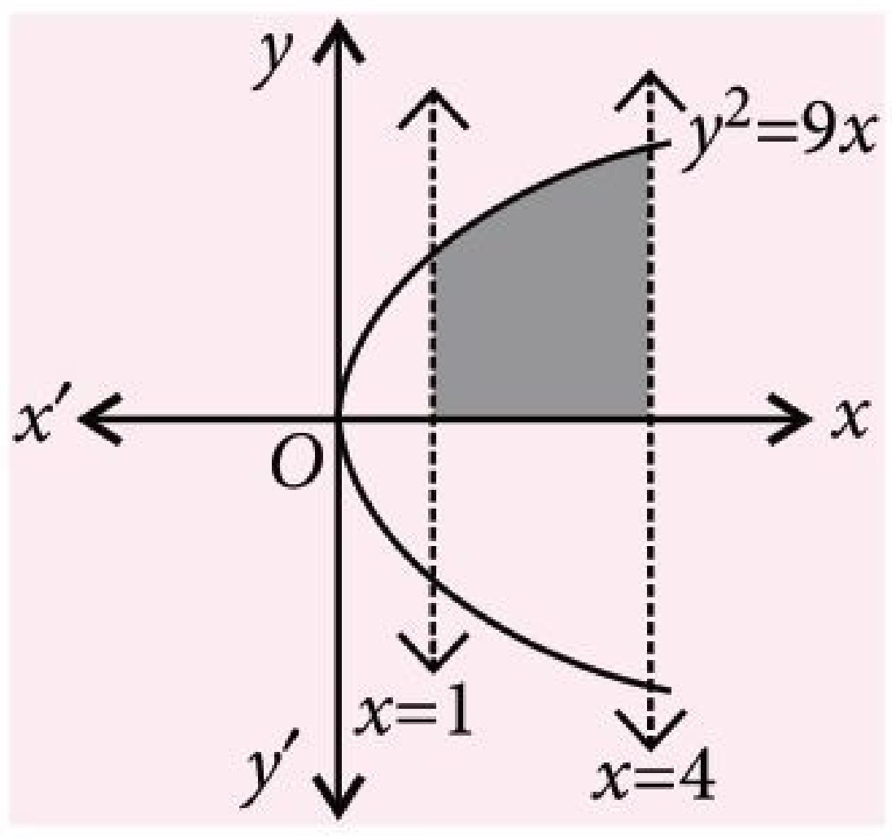
Putting $2x = y \Rightarrow 2dx = dy$

As $x\to 1 \Rightarrow y\to 2$ and $x\to 2 \Rightarrow y\to 4$

$$\therefore \quad I=\frac{1}{2}\int_2^4\left[\frac{2}{y}-\frac{2}{y^2}\right]e^y dy=\int_2^4\left[\frac{1}{y}-\frac{1}{y^2}\right]e^y dy$$

$$=\left[e^y\cdot\frac{1}{y}\right]_2^4=\frac{1}{4}e^4-\frac{1}{2}e^2=\frac{e^2}{2}\left(\frac{e^2}{2}-1\right)$$

12. We have, $y^2 = 9x$, which is a parabola.



$\therefore$ Required area

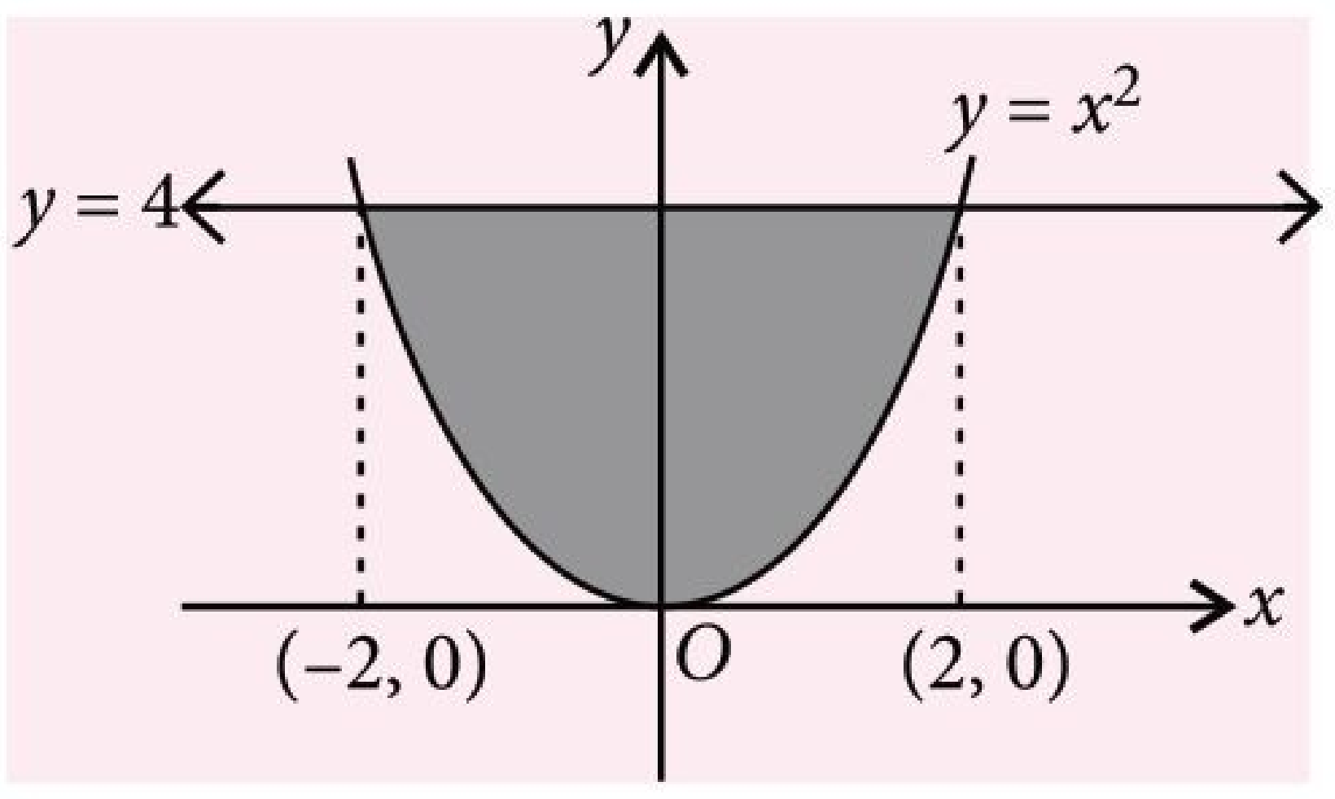
$$=\int_1^4 3\sqrt{x}\,dx=3\left[\frac{x^{3/2}}{3/2}\right]_1^4$$

$= 2(4^{3/2} - 1) = 2(8 - 1) = 14$ sq. units

OR

We have, $y = x^2$ ...(i)

and $y = 4$ ...(ii)



Required area = area of shaded region

$$=2\int_0^2(4-x^2)dx=2\left[4x-\frac{x^3}{3}\right]_0^2=\frac{32}{3}\text{sq. units}$$

13. We have, $\frac{dy}{dx}+y=\cos x-\sin x$, which is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=1$, $Q=\cos x-\sin x$

$\therefore$ I.F. $=e^{\int P dx}=e^{\int dx}=e^x$

The solution of given differential equation is

$$ye^x=\int e^x(\cos x-\sin x)dx+C$$

$$\Rightarrow ye^x=e^x\cos x+C \Rightarrow y=\cos x+Ce^{-x}$$

14. Let $I=\int\frac{x^2+x+1}{(x^2+1)(x+2)}dx$

Let $\frac{x^2+x+1}{(x^2+1)(x+2)}=\frac{Ax+B}{x^2+1}+\frac{C}{x+2}$

$\Rightarrow x^2+x+1=(Ax+B)(x+2)+C(x^2+1)$

Solving, we get

$$A=\frac{2}{5},B=\frac{1}{5}\text{ and }C=\frac{3}{5}$$

$$\therefore \quad \frac{x^2+x+1}{(x^2+1)(x+2)}=\frac{\left(\frac{2}{5}x+\frac{1}{5}\right)}{x^2+1}+\frac{\frac{3}{5}}{x+2}$$

$$=\frac{1}{5}\cdot\frac{2x+1}{x^2+1}+\frac{3}{5}\cdot\frac{1}{x+2}$$

$$\therefore \quad I=\frac{1}{5}\int\frac{2x+1}{x^2+1}dx+\frac{3}{5}\int\frac{1}{x+2}dx$$

$$=\frac{1}{5}\left[\int\frac{2x}{x^2+1}dx+\int\frac{dx}{x^2+1}\right]+\frac{3}{5}\int\frac{dx}{x+2}$$

$$=\frac{1}{5}\left[\log|x^2+1|+\tan^{-1}x\right]+\frac{3}{5}\log|x+2|+C_1$$

15. We have, $y = x^2 + 1$ ...(i)

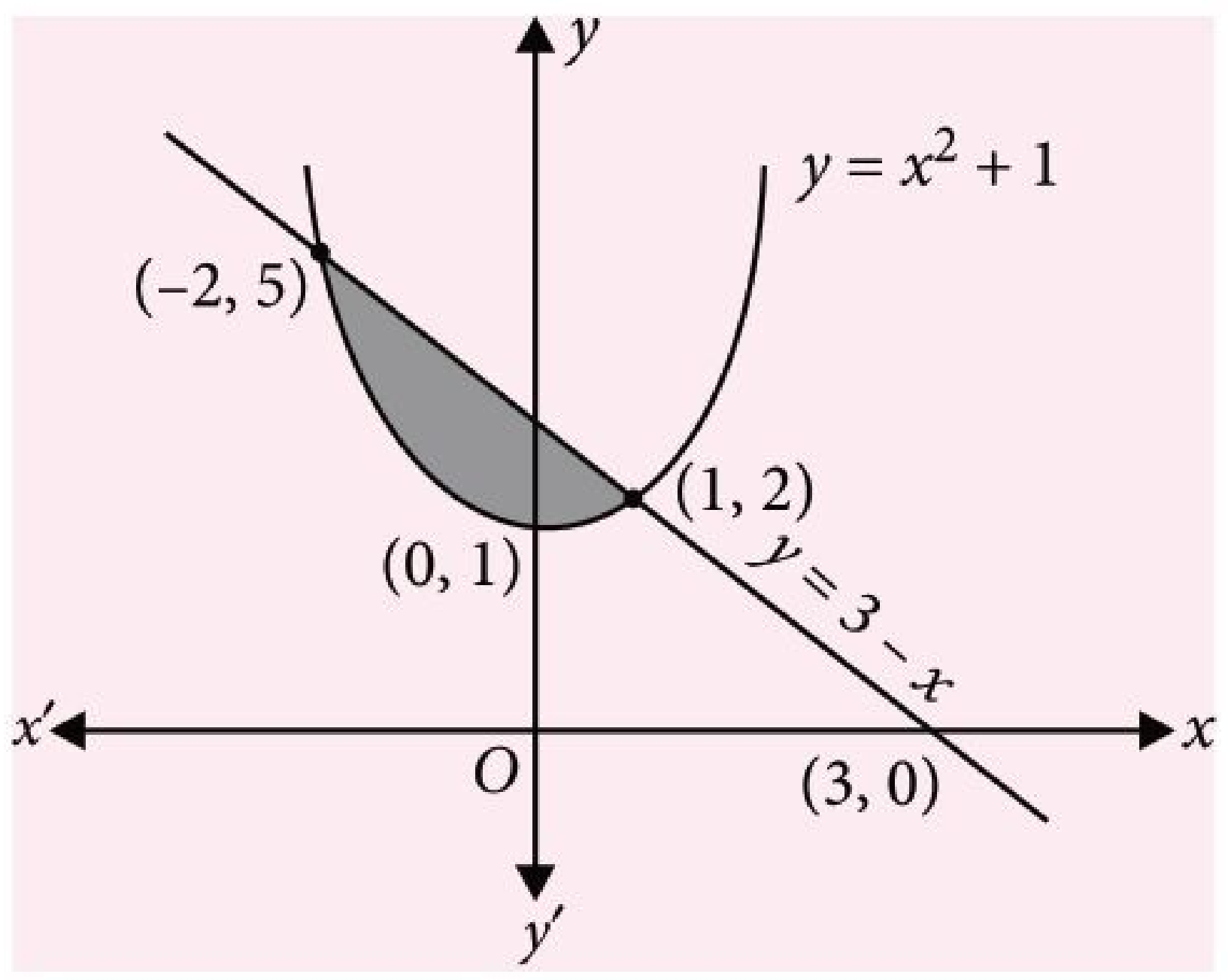
and $x + y = 3$ ...(ii)

Solving (i) and (ii), we get

$x^2 + x - 2 = 0$

$\Rightarrow$ $x = -2, 1$

$\therefore$ Required area



$$= \int_{-2}^{1} \{3 - x - (x^2 + 1)\}\, dx$$

$$= \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{1}$$

$$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right) = \frac{9}{2} \text{ sq. units}$$

16. Let $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$

$$= \int_0^{\pi} \frac{x \frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}\, dx = \int_0^{\pi} \frac{x \sin x}{1 + \sin x}\, dx \qquad \text{...(i)}$$

$$= \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1 + \sin(\pi - x)}\, dx$$

$$\left[\text{Using the property} \int_a^b f(x)dx = \int_a^b f(a + b - x)dx\right]$$

$$\Rightarrow \quad I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1 + \sin x}\, dx \qquad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi} \frac{\pi \sin x}{1 + \sin x}\, dx$$

$$\Rightarrow \quad I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{1 + \sin x}\, dx = \frac{\pi}{2}\int_0^{\pi} \frac{1 + \sin x - 1}{1 + \sin x}\, dx$$

$$= \frac{\pi}{2}\int_0^{\pi} \left(1 - \frac{1}{1 + \sin x}\right) dx; \; = \frac{\pi}{2}\left[[x]_0^{\pi} - \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x}\, dx\right]$$

$$= \frac{\pi}{2}\left[\pi - \int_0^{\pi} (\sec^2 x - \tan x \cdot \sec x)dx\right]$$

$$= \frac{\pi}{2}\left[\pi - (\tan x - \sec x)_0^{\pi}\right]$$

$$= \frac{\pi}{2}[\pi - [(0 - (-1)) - (0 - 1)]] = \frac{\pi}{2}[\pi - 2]$$

OR

Let $I = \int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

$$= \int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2 \cdot 2\sin x \cos x}}$$

$$= \frac{1}{2}\int \frac{dx}{\cos^{\left(3 + \frac{1}{2}\right)} x \cdot \sin^{\frac{1}{2}} x}$$

$$= \frac{1}{2}\int_0^{\pi/4} \frac{dx}{\cos^{\frac{7}{2}} x \cdot \tan^{\frac{1}{2}} x \cdot \cos^{\frac{1}{2}} x}$$

$$= \frac{1}{2}\int_0^{\pi/4} \frac{dx}{\cos^4 x\sqrt{\tan x}} = \frac{1}{2}\int_0^{\pi/4} \frac{\sec^4 x}{\sqrt{\tan x}}\, dx$$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

Also at $x = 0$, $t = 0$ and at $x = \frac{\pi}{4}$, $t = 1$

$$\therefore \quad I = \frac{1}{2}\int_0^1 \frac{(1 + t^2)dt}{\sqrt{t}} = \frac{1}{2}\int_0^1 (t^{-\frac{1}{2}} + t^{\frac{3}{2}})\, dt$$

$$= \frac{1}{2}\left[\frac{t^{1/2}}{1/2} + \frac{t^{5/2}}{5/2}\right]_0^1 = 1 + \frac{1}{5} = \frac{6}{5}$$

17. We have, $(1 - y^2)(1 + \log x)\, dx + 2xy\, dy = 0$

$\Rightarrow$ $(1 - y^2)(1 + \log x)\, dx = -\, 2xy\, dy$

$$\Rightarrow \quad \frac{(1 + \log x)}{x} dx = -\frac{2y}{1 - y^2} dy$$

Integrating both sides, we get

$$\frac{(1 + \log x)^2}{2} = \log|1 - y^2| + C$$

At $x = 1$, $y = 0$

$$\therefore \quad \frac{(1 + \log 1)^2}{2} = \log(1) + C \;\Rightarrow C = \frac{1}{2}$$

$$\Rightarrow \quad \frac{(1 + \log x)^2}{2} = \log|1 - y^2| + \frac{1}{2}$$

$\Rightarrow$ $(1 + \log x)^2 = 2 \log |1 - y^2| + 1$ is the required particular solution.

18. We have, $\frac{dy}{dx} - 3y\cot x = \sin 2x$ ...(i)

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = -3 \cot x$ and $Q = \sin 2x$.

$\therefore$ $\text{I.F.} = e^{\int P dx} = e^{-3\int \cot x dx}$

$= e^{-3\log|\sin x|} = |\sin^{-3} x|$

$$\therefore \quad y \cdot \sin^{-3} x = \int \sin 2x \cdot \sin^{-3} x\, dx + C$$

$$\Rightarrow \quad \frac{y}{\sin^3 x} = \int \frac{2\sin x \cos x}{\sin^3 x} dx + C$$

$= \int \frac{2\cos x}{\sin^2 x} dx + C$ (Put $\sin x = t \Rightarrow \cos x\, dx = dt$)

$$= 2\int\frac{dt}{t^2}+C = -\frac{2}{t}+C=-\frac{2}{\sin x}+C$$

$$\Rightarrow\ y=-2\sin^2 x+C\sin^3 x \qquad \text{...(ii)}$$

Put $x=\frac{\pi}{2}, y=2$ in (ii), we get

$2 = -2\cdot1 + C\cdot 1 \Rightarrow C = 4$

∴ $y = 4\sin^3 x - 2\sin^2 x$ is the required particular solution.

OR

We have, $\left(x\sin^2\left(\frac{y}{x}\right)-y\right)dx+x\,dy=0$

$$\Rightarrow\ \frac{dy}{dx}+\sin^2\left(\frac{y}{x}\right)-\frac{y}{x}=0 \qquad \text{...(i)}$$

Put $y = vx \Rightarrow \frac{dy}{dx}=v\cdot1+x\frac{dv}{dx}$

∴ (i) becomes

$$v+x\frac{dv}{dx}+\sin^2 v-v=0$$

$$\Rightarrow\ x\frac{dv}{dx}+\sin^2 v=0 \ \Rightarrow\ \operatorname{cosec}^2 v\,dv+\frac{dx}{x}=0$$

Integrating both sides, we get

$$\int\operatorname{cosec}^2 v\,dv+\int\frac{dx}{x}=C \ \Rightarrow\ -\cot v+\log x=C$$

$$\Rightarrow\ -\cot\left(\frac{y}{x}\right)+\log x=C \qquad \text{...(ii)}$$

Put $x = 1$, $y = \pi/4$ in (ii), we get

$$-\cot\frac{\pi}{4}+\log 1=C \ \Rightarrow\ C=-1$$

∴ $-\cot\left(\frac{y}{x}\right)+\log x+1=0$ is the required particular solution.

19. Let $I=\int\frac{3x+5}{x^3-x^2-x+1}dx$

Here, $x^3 - x^2 - x + 1 = x^2(x - 1) - 1(x - 1)$

$= (x^2 - 1)(x - 1) = (x - 1)(x + 1)(x - 1) = (x - 1)^2(x + 1)$

Let $\frac{3x+5}{x^3-x^2-x+1}=\frac{3x+5}{(x-1)^2(x+1)}$

$$=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+1}$$

$\Rightarrow\ 3x + 5 = A(x - 1)(x + 1) + B(x + 1) + C(x - 1)^2$

Putting $x = 1, -1, 0$, we get

$A=-\frac{1}{2}$; $B = 4$; $C=\frac{1}{2}$

$$\therefore\ \frac{3x+5}{x^3-x^2-x+1}=-\frac{1}{2}\cdot\frac{1}{x-1}+\frac{4}{(x-1)^2}+\frac{1}{2}\cdot\frac{1}{x+1}$$

Integrating both sides, we get

$$I=-\frac{1}{2}\int\frac{dx}{x-1}+4\int\frac{dx}{(x-1)^2}+\frac{1}{2}\int\frac{dx}{x+1}$$

$$=-\frac{1}{2}\log|x-1|-\frac{4}{x-1}+\frac{1}{2}\log|x+1|+C_1$$

$$=\frac{1}{2}\log\left|\frac{x+1}{x-1}\right|-\frac{4}{x-1}+C_1$$

OR

$$\text{Let } I=\int_0^{\pi}\frac{x\,dx}{a^2\cos^2 x+b^2\sin^2 x} \qquad \text{...(i)}$$

$$\Rightarrow\ I=\int_0^{\pi}\frac{(\pi-x)dx}{a^2\cos^2(\pi-x)+b^2\sin^2(\pi-x)}$$

$$\left[\text{Using } \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

$$\Rightarrow\ I=\int_0^{\pi}\frac{(\pi-x)dx}{a^2\cos^2 x+b^2\sin^2 x} \qquad \text{...(ii)}$$

Adding (i) and (ii), we get

$$I=\frac{\pi}{2}\int_0^{\pi}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$$

Let $f(x)=\frac{1}{a^2\cos^2 x+b^2\sin^2 x}$

$$\Rightarrow\ f(\pi-x)=\frac{1}{a^2\cos^2(\pi-x)+b^2\sin^2(\pi-x)}$$

$$\Rightarrow\ f(\pi-x)=\frac{1}{a^2\cos^2 x+b^2\sin^2 x}=f(x)$$

$$\left[\text{Using } \int_0^{2a} f(x)dx=2\int_0^{a} f(x)\,dx, \text{ if } f(2a-x)=f(x)\right]$$

$$\therefore\ I=\frac{\pi}{2}\left(2\int_0^{\pi/2}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}\right)$$

$$\Rightarrow\ I=\pi\int_0^{\pi/2}\frac{\sec^2 x\,dx}{a^2+b^2\tan^2 x}$$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

Also at $x = 0$, $t = \tan 0 = 0$

And at $x=\frac{\pi}{2}$, $t=\tan\frac{\pi}{2}=\infty$

$$\therefore\ I=\pi\int_0^{\infty}\frac{dt}{a^2+b^2t^2}\Rightarrow I=\frac{\pi}{b^2}\int_0^{\infty}\frac{dt}{\left(\frac{a}{b}\right)^2+t^2}$$

$$\Rightarrow\ I=\frac{\pi}{b^2}\left[\frac{b}{a}\tan^{-1}\left(\frac{bt}{a}\right)\right]_0^{\infty}$$

$$\Rightarrow\ I=\frac{\pi}{ab}\left[\tan^{-1}\infty-\tan^{-1}0\right]=\frac{\pi^2}{2ab}$$

◆◆

MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 7 : Definite Integrals and Application of Integrals

Time Taken : 60 Min.

Only One Option Correct Type

1. If $3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-5\ \forall\ x\neq 0$, then $\int\limits_1^2 f(x)\,dx=$

(a) $\frac{3\ln 2+4}{5}$ (b) $\frac{3\ln 2-4}{5}$

(c) $\frac{3\ln 2-8}{5}$ (d) $\frac{3\ln 2+8}{5}$

2. The value of $\lim\limits_{x\to\frac{\pi}{4}}\frac{\int_2^{\sec^2 x} f(t)\,dt}{x^2-\frac{\pi^2}{16}}$ is

(a) $\frac{8}{\pi}f(2)$ (b) $\frac{2}{\pi}f(2)$

(c) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ (d) $4f(2)$

3. The area bounded by the curve $C: y=\tan x$, tangent drawn to C at $x=\frac{\pi}{4}$ and the x-axis (in sq. units) is

(a) $\ln\sqrt{2}-\frac{1}{2}$ (b) $\ln\sqrt{2}-\frac{1}{4}$

(c) $\ln\sqrt{2}+\frac{1}{2}$ (d) $\ln\sqrt{2}+\frac{1}{4}$

4. The area of the region between the curves $y=\sqrt{\frac{1+\sin x}{\cos x}}$ and $y=\sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines $x=0$ and $x=\frac{\pi}{4}$ (in sq. units) is

(a) $\int\limits_0^{\sqrt{2}-1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

(b) $\int\limits_0^{\sqrt{2}-1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(c) $\int\limits_0^{\sqrt{2}+1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$

(d) $\int\limits_0^{\sqrt{2}+1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

5. The larger area bounded by $y^2=4x$ and $x^2+y^2-2x-3=0$ (in sq. units) is

(a) $2\pi-\frac{4}{3}$ (b) $2\pi+\frac{4}{3}$

(c) $2\pi+\frac{8}{3}$ (d) $2\pi+\frac{2}{3}$

6. The value of $\int_0^1\frac{dx}{\left(1+x^{\frac{1}{4}}\right)\sqrt{\sqrt{x}-x}}$ is

(a) $2\pi-8$ (b) $2\pi-4$

(c) $8-2\pi$ (d) $4+2\pi$

One or More than One Option(s) Correct Type

7. The value of definite integral $\int\limits_0^{2\pi} x\log_e\left(\frac{3+\cos x}{3-\cos x}\right)dx$ is

(a) $\pi\int\limits_0^{2\pi}\log_e\left(\frac{3+\cos x}{3-\cos x}\right)dx$

(b) $2\pi\int\limits_0^{\pi}\log_e\left(\frac{3+\cos x}{3-\cos x}\right)dx$

(c) 0

(d) $2\pi\int\limits_0^{\pi}\log_e\left(\frac{3-\cos x}{3+\cos x}\right)dx$

8. If $A=\int_0^{\pi}\frac{\sin x}{\sin x+\cos x}dx$ and $B=\int_0^{\pi}\frac{\sin x}{\sin x-\cos x}dx$, then

(a) $A = B$
(b) A and B both are rational
(c) A and B both are irrational
(d) A is period of $|\sin x|$

9. Area bounded by curves $y=\frac{x^2}{4a}$ and $y=\frac{8a^3}{x^2+4a^2}$ is

(a) $\frac{1}{3}(6\pi-4)$, if $a = 1$
(b) $\frac{1}{3}(4\pi+3)$, if $a = 1$
(c) $\frac{4}{3}(6\pi-4)$, if $a = 2$
(d) $\frac{1}{3}(2\pi+3)$, if $a = 1$

10. The area bounded by the parabola $y = x^2 - 7x + 10$ and x-axis equals

(a) area bounded by $y = -x^2 + 7x - 10$
(b) $\frac{1}{6}$ sq. units
(c) $\frac{5}{6}$ sq. units (d) $\frac{9}{2}$ sq. units

11. If $I_1=\int_0^1 2^{x^2}dx$, $I_2=\int_0^1 2^{x^3}dx$, $I_3=\int_1^2 2^{x^2}dx$ and $I_4=\int_1^2 2^{x^3}dx$, then

(a) $I_1 > I_2$ (b) $I_1 < I_2$
(c) $I_3 < I_4$ (d) $I_3 = I_4$

12. If $I_n=\int_0^{\pi/4}\tan^n x\,dx$, then

(a) $I_7+I_5=\frac{1}{6}$ (b) $I_{10}+I_8=\frac{1}{9}$
(c) $I_8-I_{12}=\frac{2}{99}$ (d) $I_{12}+2I_{10}+I_8=\frac{20}{99}$

Comprehension Type

Paragraph for Q. No. 13 to 15

Differentiate I w.r.t. the parameter within the sign of integrals taking variable of the integrand as constant. Now, evaluate the integral so obtained as usual as a function of the parameter and then integrate the result to get I. Constant of integration can be computed by giving some arbitrary values to the parameter and the corresponding value of I.

13. The value of $\int_0^1\frac{x^a-1}{\log x}dx$ is

(a) $\log(a - 1)$ (b) $\log(a + 1)$
(c) $a\log(a + 1)$ (d) None of these

14. The value of $\int_0^{\pi/2}\log(\sin^2\theta+k^2\cos^2\theta)d\theta$, where $k\ge 0$, is

(a) $\pi\log(1 + k) + \pi\log 2$
(b) $\pi\log(1 + k)$
(c) $\pi\log(1 + k) - \pi\log 2$
(d) $\pi\log(1 + k) - \log 2$

15. If $\int_0^{\pi}\frac{dx}{(a-\cos x)}=\frac{\pi}{\sqrt{a^2-1}}$, then the value of $\int_0^{\pi}\frac{dx}{(\sqrt{10}-\cos x)^3}$ is

(a) $\frac{\pi}{81}$ (b) $\frac{7\pi}{81}$
(c) $\frac{7\pi}{162}$ (d) $\frac{7\pi}{8}$

Matrix Match Type

16. Match the following :

	Column-I		**Column-II**
P.	If $I_1=\int_0^1 x^4(1-x)^5dx$ and $I_2=\int_0^1 x^5(1-x)^5dx$, then $\frac{I_2}{I_1}=$	1.	$\frac{1}{2}$
Q.	If $I_1=\int_0^{\pi}\cos^4 x\sin^6 x\,dx$ and $I_2=\int_0^{2\pi}\cos^4 x\sin^4 x\,dx$, then $\frac{I_1}{I_2}=$	2.	$-\frac{1}{3}$
R.	If $f(x)=\frac{\sin^2 x}{(1-e^{-3x})\sin^2 2x}$, then $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}f(x)dx=$	3.	$\frac{1}{4}$

S.	$\int_0^{\frac{\pi}{2}}\left(\sqrt{\sin x}+\sqrt{\cos x}\right)^{-4} dx =$	4.	$\frac{5}{11}$

	P	Q	R	S
(a)	1	2	3	4
(b)	2	3	4	1
(c)	4	3	3	2
(d)	3	2	1	4

Numerical Answer Type

17. The value of

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (f(x)+f(-x))(g(x)-g(-x))dx = \_\_\_\_\_\_\_\_\_.$$

18. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x} dx$ = ________.

19. The area of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is ________.

20. Let $F(x) = \int_{x}^{x+3} 2t(5-t)dt$. If M is the maximum value of $F(x)$, then the sum of the digits of M is equal to ________.

◆ ◆

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SELF CHECK

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No. of questions correct ……

Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
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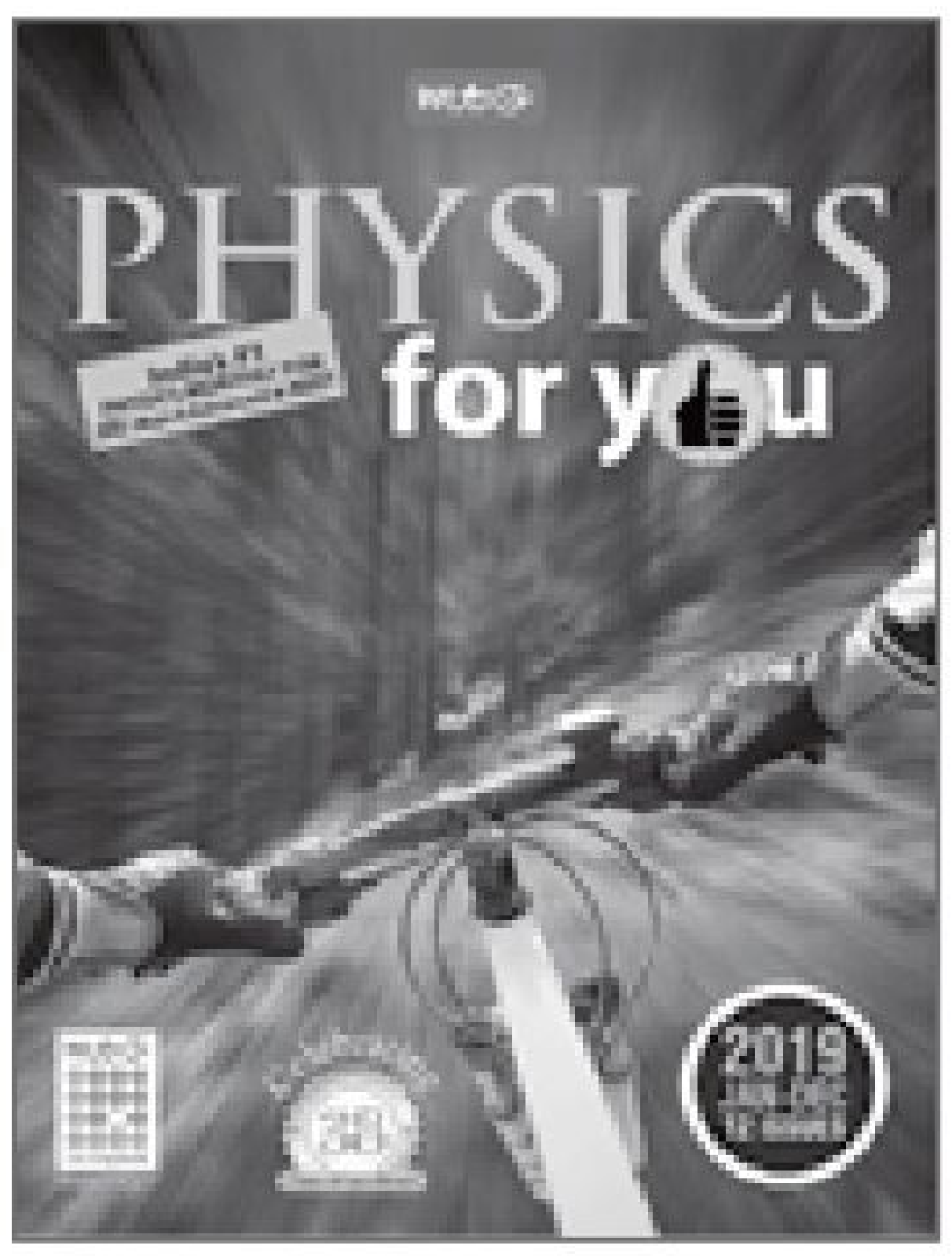
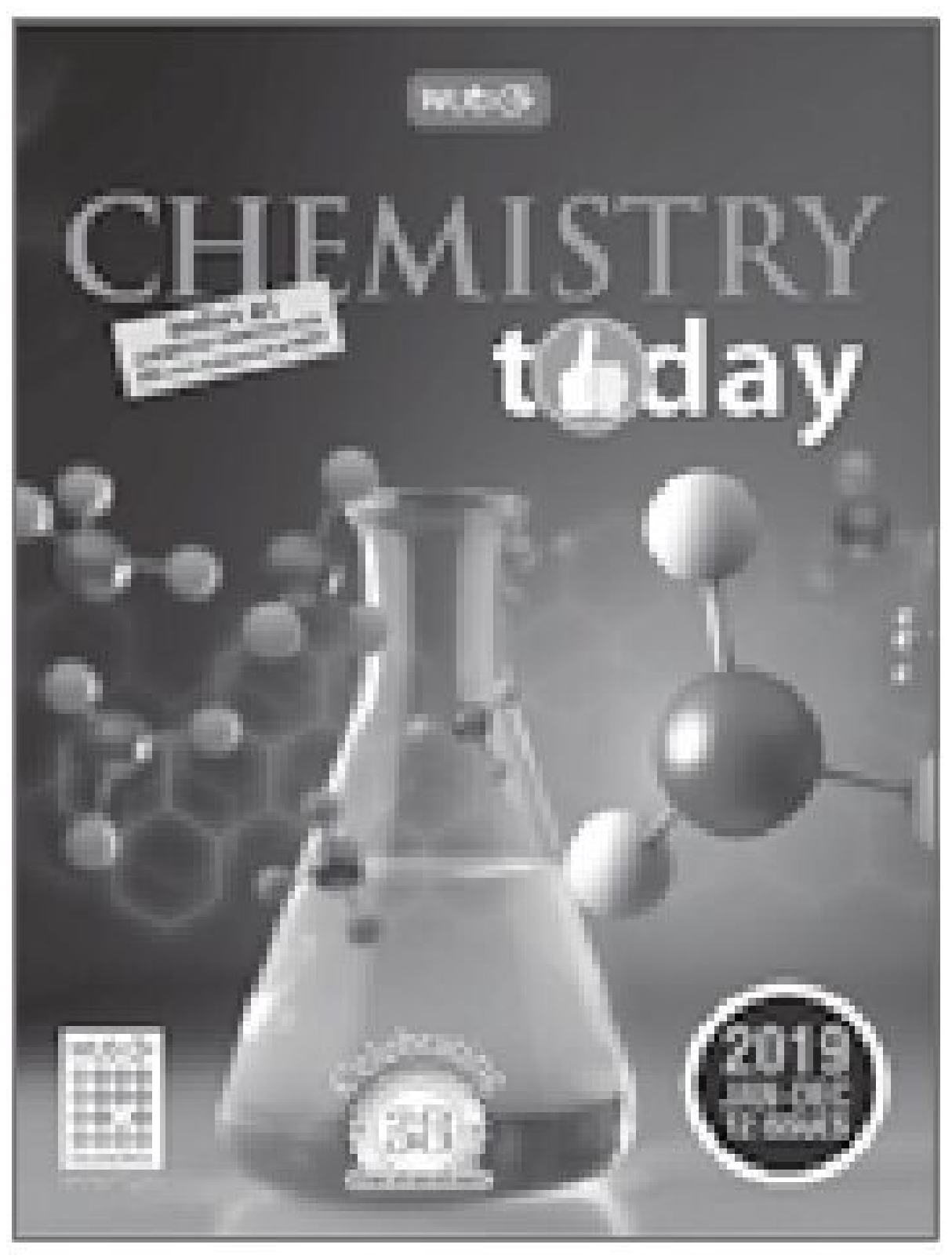
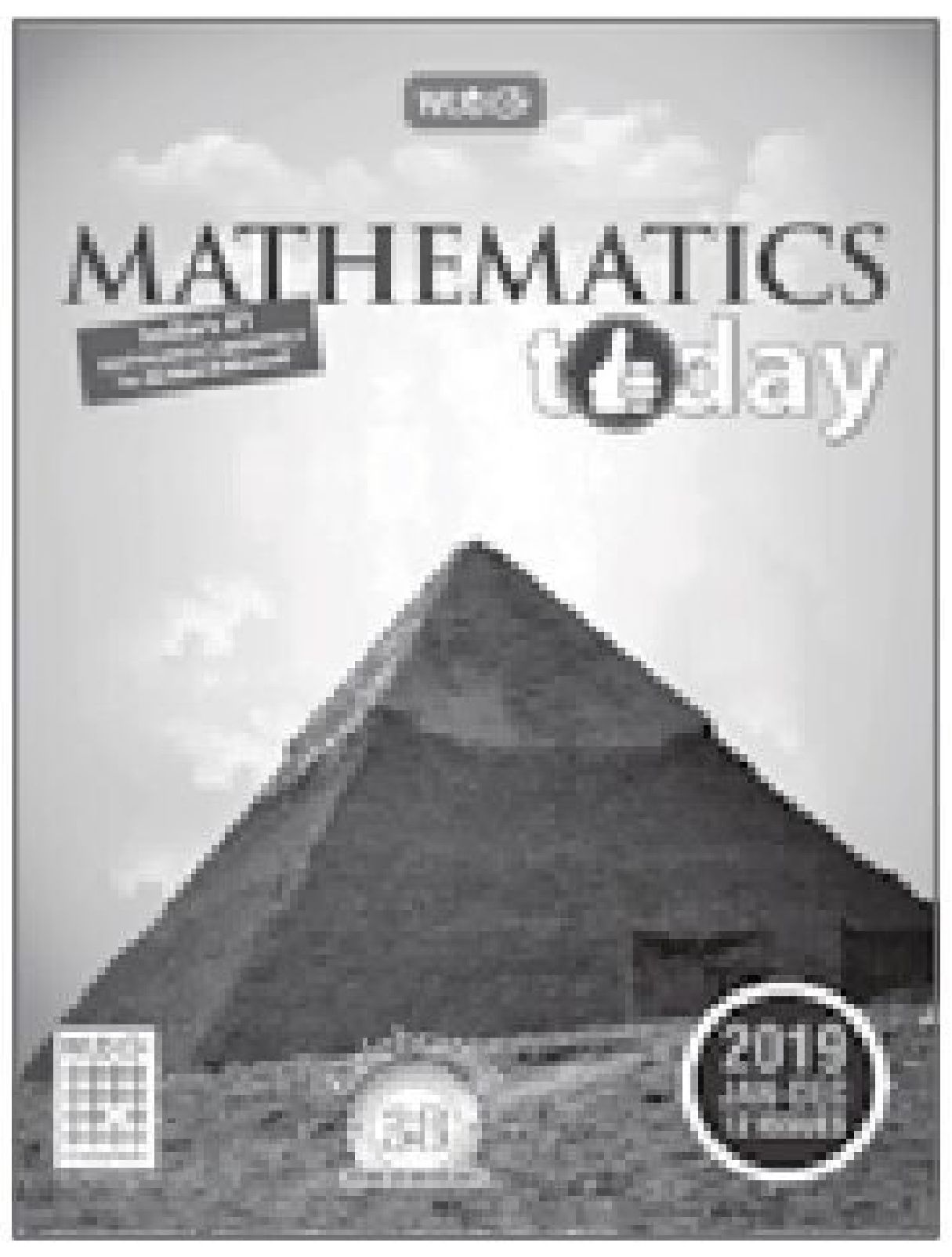
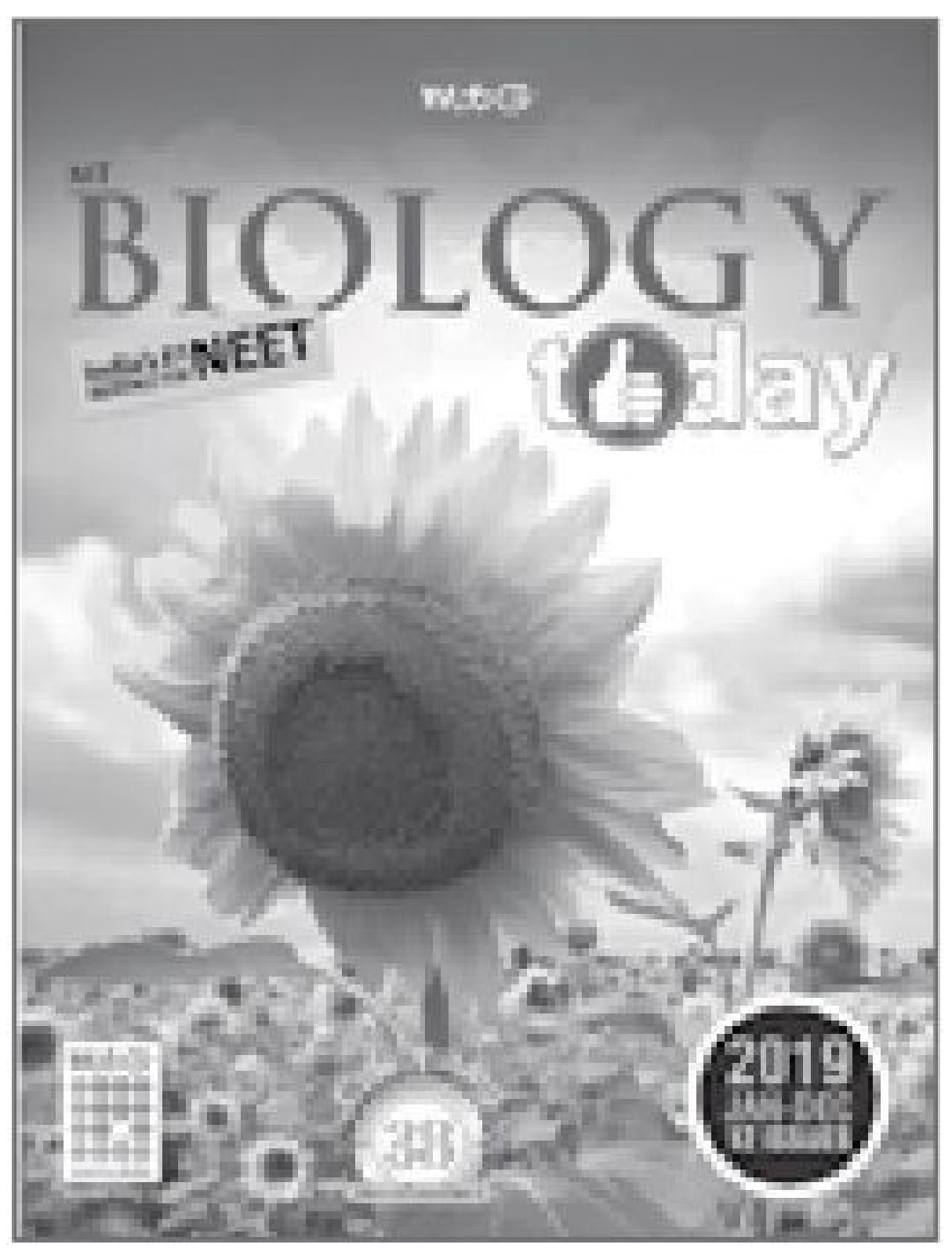
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.









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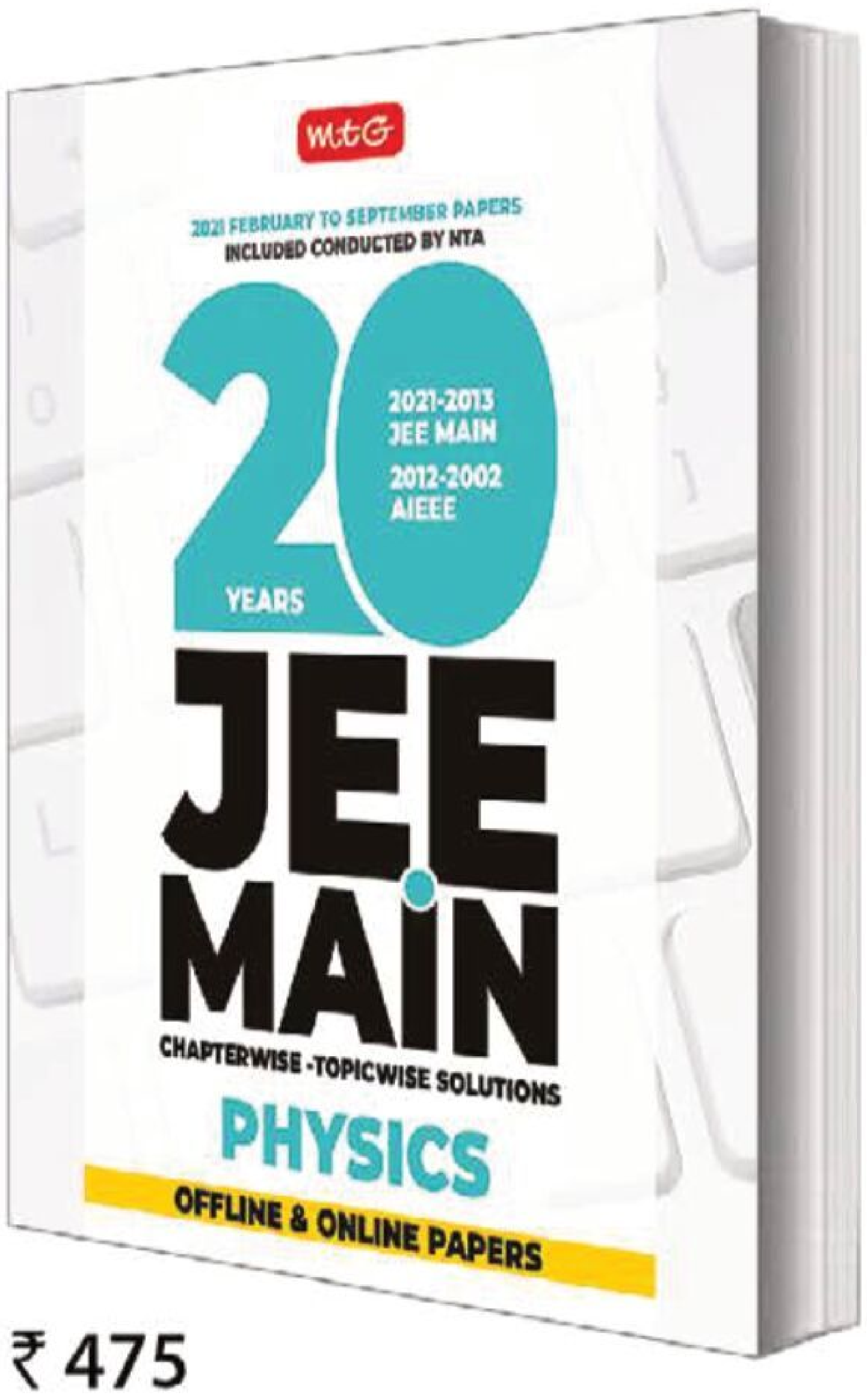
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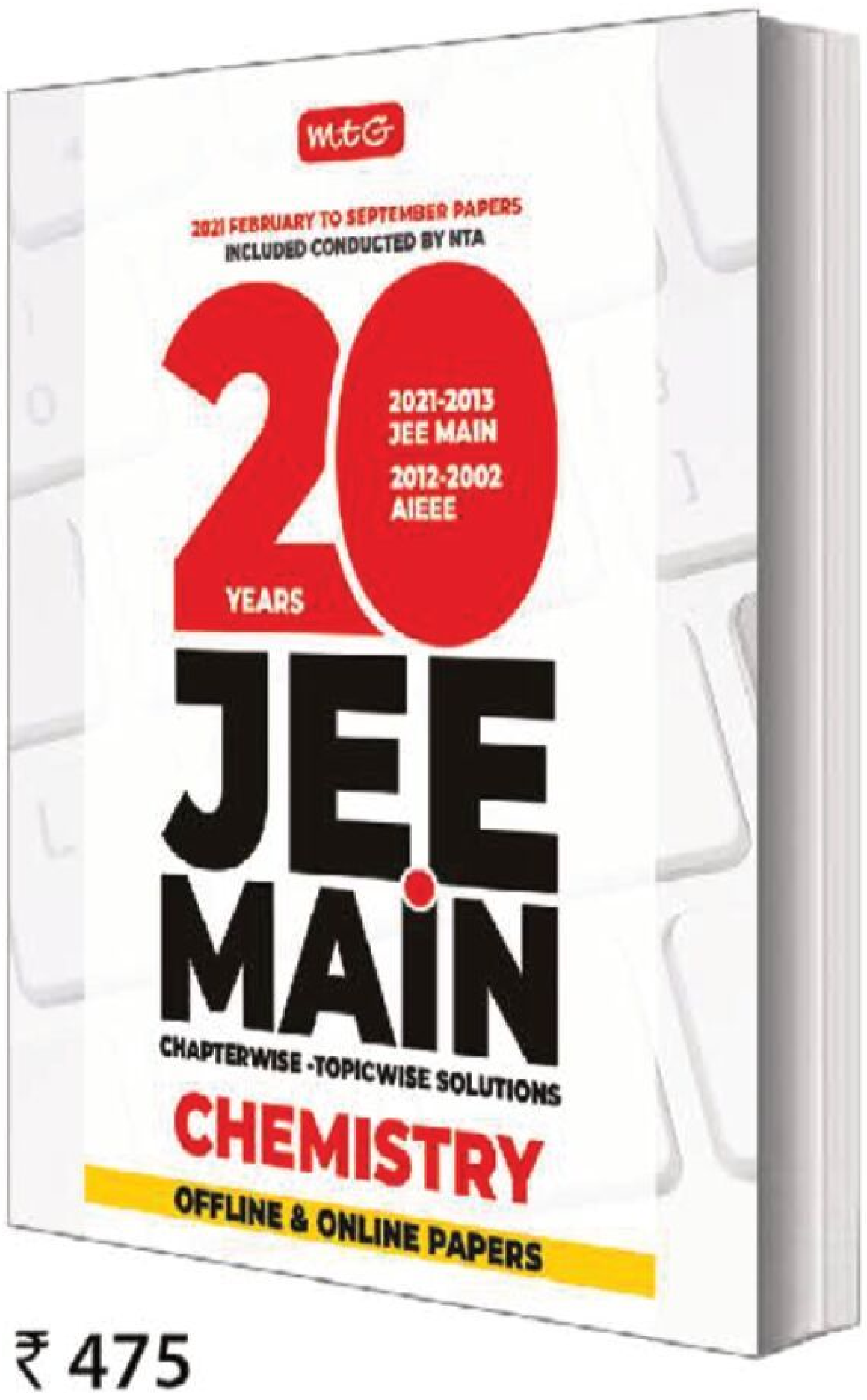
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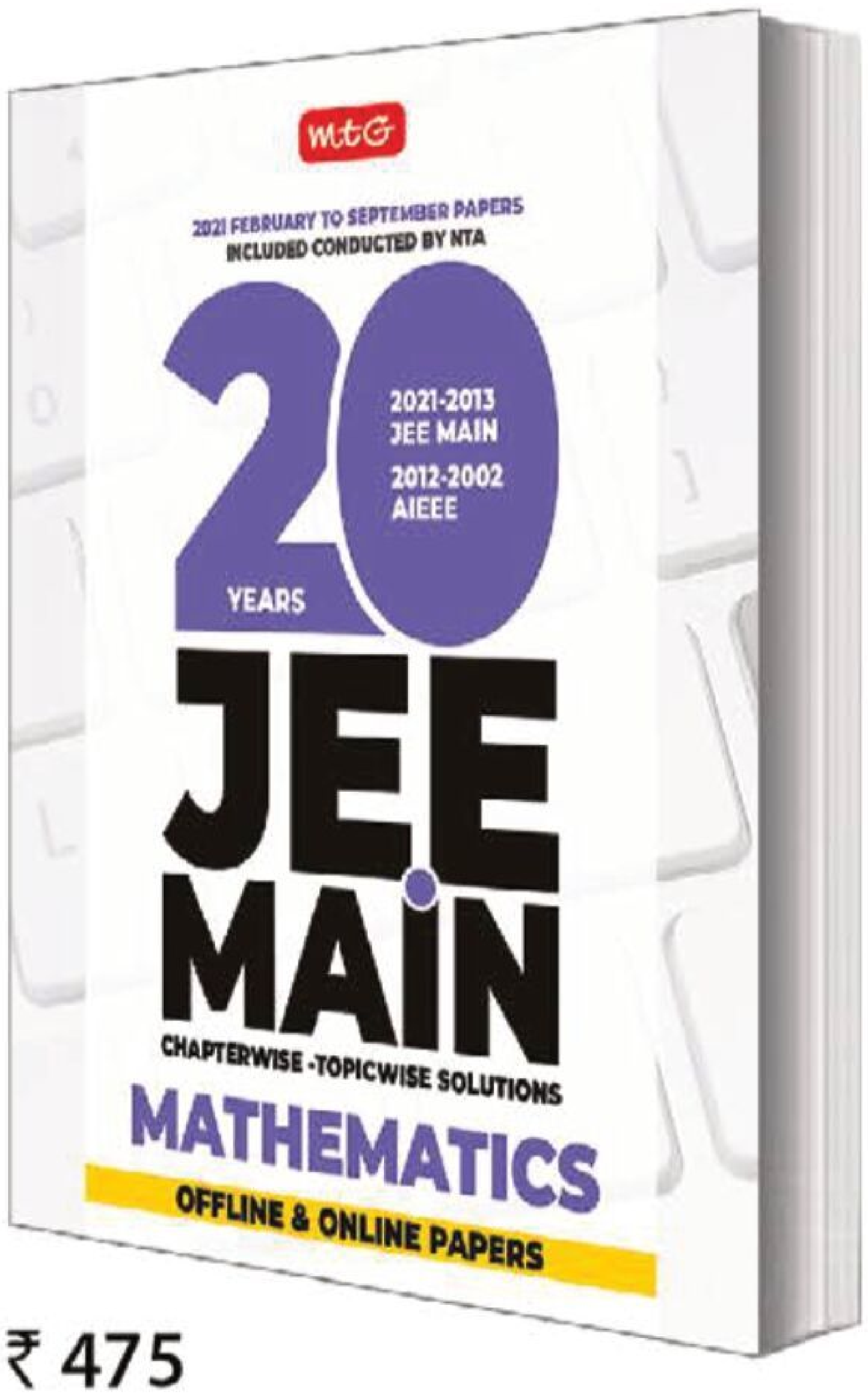
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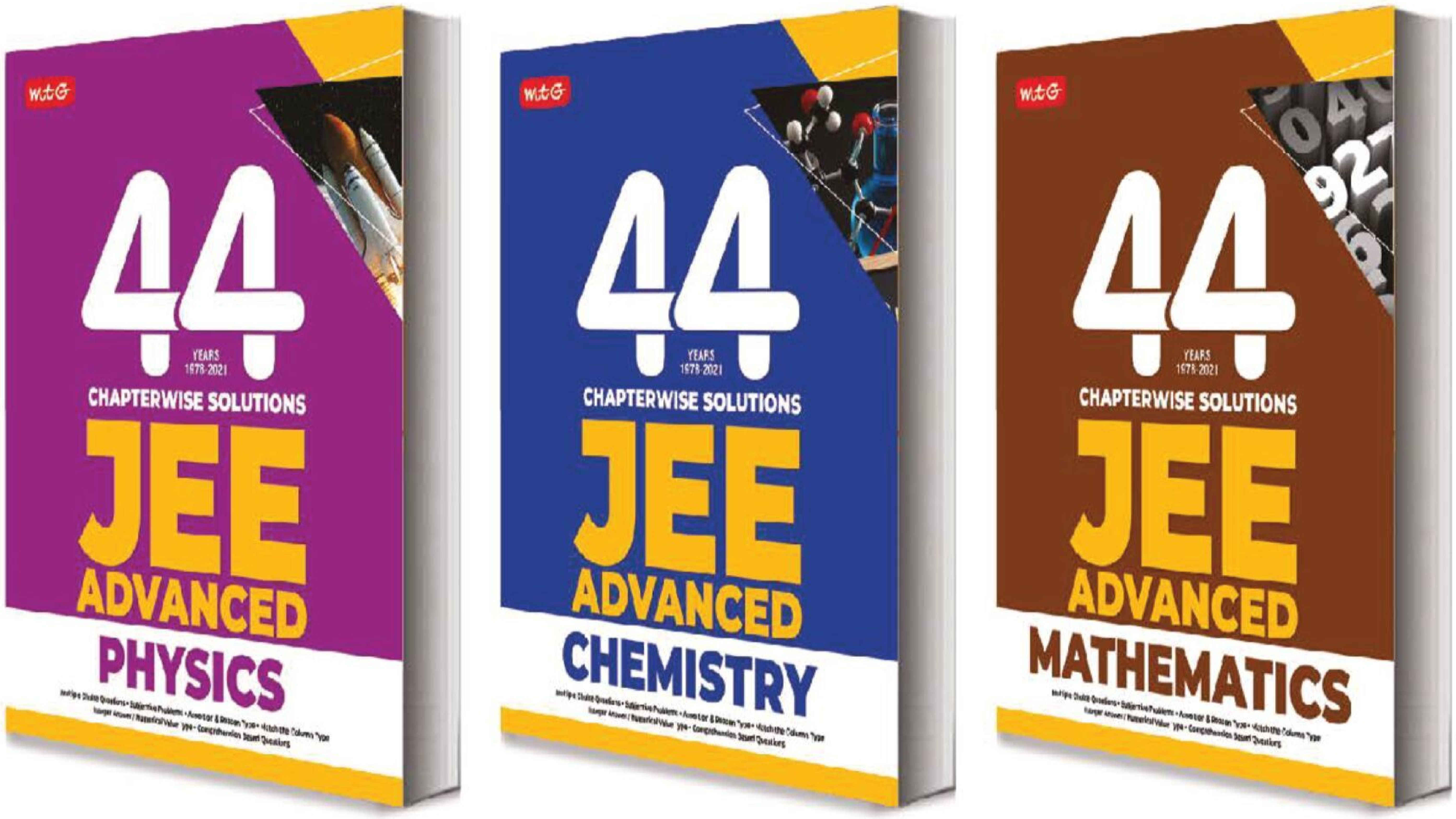
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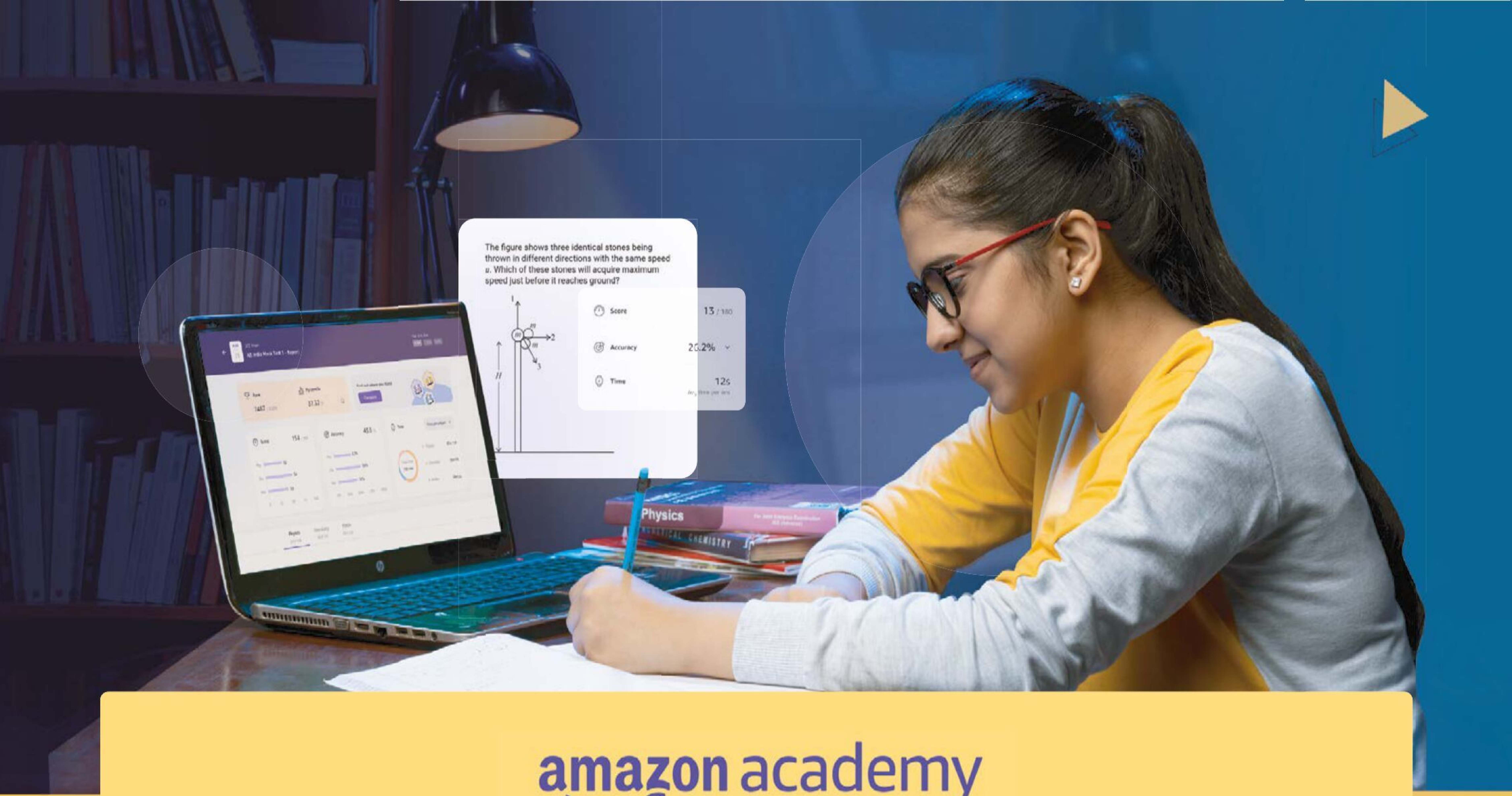

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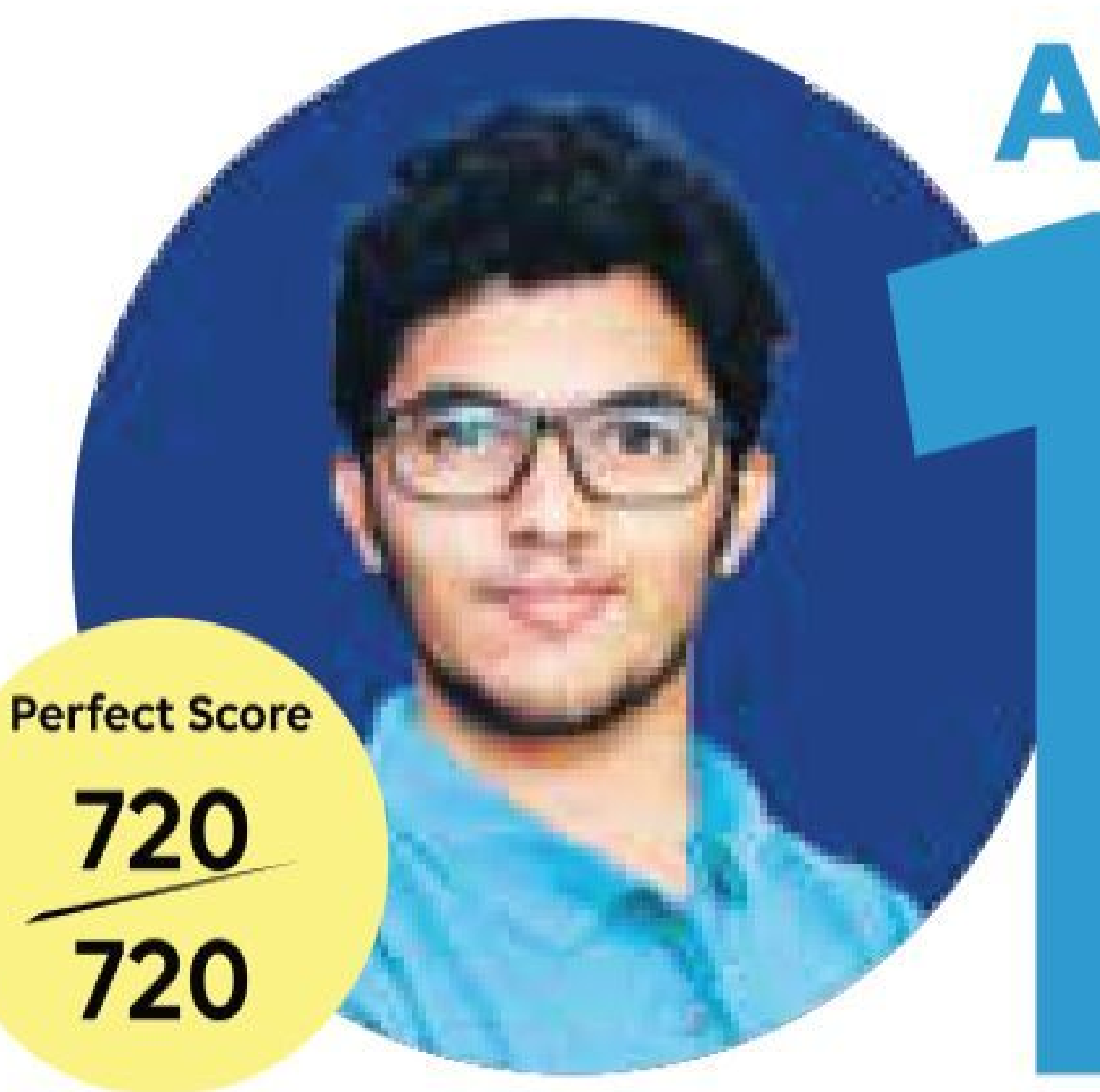

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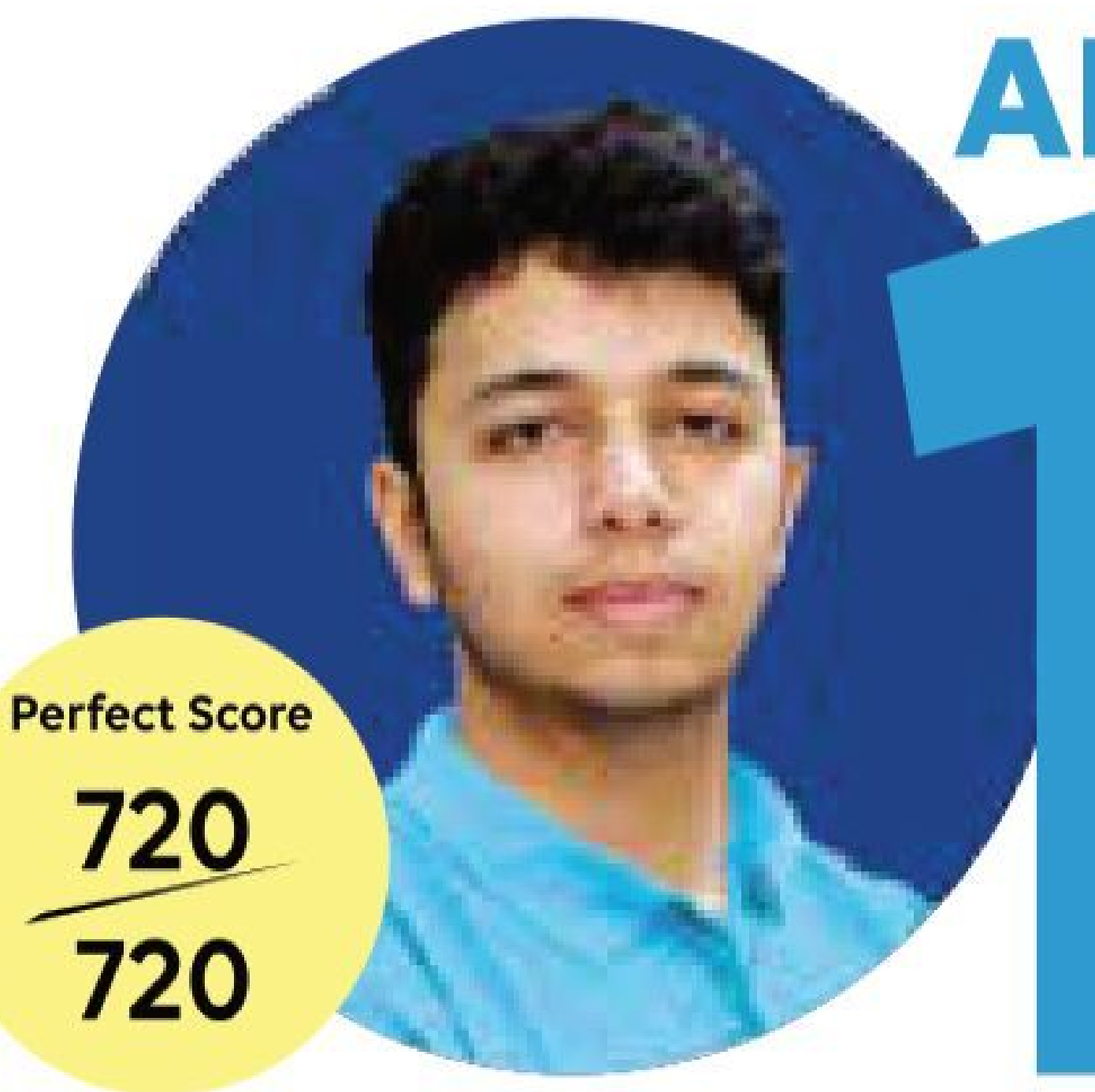

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